Assessment of Second Piola-Kirchhoff Stress Tensor in Laminated Piezoelectric Structures through SaS Shell Formulation

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ABSTRACT

The paper presents a robust nonlinear exact geometry or geometrically exact (GeX) four-node laminated piezoelectric solid-shell element using the method of sampling surfaces (SaS) [1, 2]. The term GeX reflects the fact that the parametrization of the middle surface is known and, therefore, coefficients of the first and second fundamental forms and Christoffel symbols are taken exactly at element nodes. The SaS method is based on choosing inside the *n*th layer I_n SaS parallel to the middle surface in order to introduce the displacements and electric potentials of these surfaces as basic shell unknowns. Such choice of unknowns with the consequent use of Lagrange polynomials of degree $I_n - 1$ in assumed approximations of displacements, electric potential, strains and electric field through the layer thickness yields the efficient higher-order piezoelectric shell formulation. The inner SaS are located inside each layer at Chebyshev polynomial nodes that makes it possible to minimize uniformly the error due to Lagrange interpolation. As a result, the GeX piezoelectric shell formulation can be applied to obtaining the numerical solutions for laminated piezoelectric shells, which asymptotically approach the 3D exact solutions of electroelasticity as the number of SaS tends to infinity.

The SaS shell formulation utilizes the objective Green-Lagrange strain tensor that exactly represents the arbitrarily large rigid-body motions of a shell in any curvilinear coordinate system. This permits one to calculate the transverse components of the second Piola-Kirchhoff stress tensor in piezoelectric doubly-curved shells with a high accuracy. The developed GeX solid-shell element is based on the hybrid-mixed method that allows the use of loading increments, which are much larger than possible with the displacement-based solid-shell element formulations. The tangent stiffness matrix is evaluated through effective 3D analytical integration. The developed GeX solid-shell element naturally generalizes the 9-parameter solid-shell elements [3, 4] in which bottom, middle and top surfaces are accepted as SaS.

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