Carbon nanotubes modelled as size-dependent nanobeams for mass sensing applications

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Abstract. In this research, we investigate the behavior, response, and limits of applicability for considering carbon nanotube (CNT) -based mechanical resonators for nanoscale mass sensing applications. Following their initial discovery and investigations on unique material, chemical, thermal, and electrical properties, CNTs are considered an ideal resonator candidate due to their high resonant frequency and stiffness. This mechanical property itself lends them to join the classification of a unique set of structures. To this end, we work to develop reduced-order models for CNT-based resonators capable of detecting the mass and geometry of attached nanoparticles. Thus, we propose a nonlocal, continuum-based model of a carbon nanotube-based mechanical resonator that relies on predicting the mass of the deposited particle through the inherent shift in the frequency before and after deposition. Depending on the aspect ratio, number of walls, and wall thickness, the CNT may be modeled as an elastic Euler Bernoulli beam, Timoshenko beam, or as a thin-walled shell depending on the need to account for the effects of bending, transverse shear deformation, rotary inertia, etc. The model should also consider von Kármán geometric nonlinearity due to mid-plane stretching for end-constrained systems. We utilized Eringen's nonlocal elasticity theory to account for long-range interatomic interactions; *i.e.,* the size dependent effects that become influential at the nanoscale. Our developed model can account for an atomic mass deposited at any arbitrary location along the longitudinal axis of the CNT. Additionally, the effects of the geometry of the particles are considered through inertial terms. The nonlinear, nonlocal governing equations and compatibility conditions are derived using Hamilton's Principle, then are non-dimensionalized for further analysis. Neglecting the nonlinear terms, we determined the linear natural frequencies and associated mode shapes from an eigenvalue problem analysis. We solved our equations analytically for the vibration frequencies of the CNT beam using various boundary conditions, such as clamped-clamped, clamped-hinged, and hinged-hinged. We then performed a parametric study to determine the effects of different variables on the frequency shift and sensitivity of the CNT to the nanoscale object's mass. These variables include the CNT length to diameter ratio, shear factor, nonlocal parameter, and landing location and effective geometry of the nanoscale object along the longitudinal axis of the CNT. Our preliminary study shows that shear deformation and rotary inertia effects will be significant for short, stout beams and at relatively high frequencies of vibration. Our derived model and analysis can be utilized by other researchers in this field who are conducting experiments at nano-scale. Our obtained results can be used as a good reference, and can hopefully help to reduce the need to computationally expensive high-fidelity models at nano-scale, or costly experiments that are difficult to conduct.