# Transported mesh-free methods for tackling the curse of dimensionality

**Philippe G. LeFloch*** **and Jean-Marc Mercier****[,1]

* Laboratoire Jacques-Louis Lions and CNRS, Sorbonne Université,
4 Place Jussieu, 75252 Paris, France. Email: contact@philippelefloch.org

** MPG-Partners, 136 Boulevard Haussmann, 75008 Paris, France.
Email: jean-marc.mercier@gmail.com
[1] speaker for this conference

## ABSTRACT

We introduce a new numerical strategy for solving a broad class of partial differential equations (PDEs), which we refer to as the **transported mesh-free methods** (TMFM). The development of the TMFM stemmed from the Lagrangian mesh-free algorithms which are commonly used in fluid dynamics — however, a major difference is that TMFM can handle both advection and diffusion terms. We rely on the notion of "transport" which is central in the mathematical theory of optimal transport and provides us with one key ingredient of our approach.

The strategy was first presented in [1], in which we investigated numerical applications in mathematical finance, and is nowadays used for industrial applications. As standard mesh-free method, the TMFM rely on the choice of a kernel and, interestingly enough, as Lagrangian mesh-free methods, TMFM can be interpreted in the context of neural networks, addressing also deep learning techniques for solving PDEs.

In particular, in [1] we presented numerical experiments for computing convection-diffusion PDEs in arbitrarily high dimensions. We recall that solving high-dimensional PDEs is still considered today as an open problem, referred to as the *curse of dimensionality* —an expression coined by R. Bellman in 1961.

In this talk, we will present the TMFM strategy. We will apply it to the Fokker-Plank equation, that is, a nonlinear convection-diffusion equation describing the time evolution of a probability density measure. The numerical scheme will be described and its properties will be illustrated numerically —for definiteness on a two-dimensional problem.

We will also discuss the convergence properties of our numerical schemes, and will present a new error estimate recently established in [2]. Our convergence results are optimal and in particular depend —in a way we can analyze quantitatively— upon the choice of the kernel, the dimension of the problem, and the regularity of the solutions under consideration. Consequently, we are able to characterize those kernels that can be considered as essentially breaking the curse of dimensionality or not. We provide numerical simulations using kernels illustrating both regimes.

## REFERENCES

[1] LeFloch P.G. and Mercier J.M., *A new method for solving Kolmogorov equations in mathematical finance*, Comptes Rendus Acad. Sc. Paris (C.R.A.S.) 355 (2017), 680–686.

[2] LeFloch P.G. and Mercier J.M., *A sharp error integration estimate for mesh-less methods in arbitrary dimensions*, Preprint 2019.