Particle Simulation of Space-Fractional Diffusion Equations

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ABSTRACT

This work explores different particle-based approaches to the simulation of space-fractional diffusion equations in unbounded domains. We rely on smooth particle approximations, and consider five different methods for estimating the fractional diffusion term. The first method is based on a direct differentiation of the particle representation, following the Riesz definition of the fractional derivative, and results in a non-conservative scheme. Three methods follow the particles strength exchange (PSE) methodology and are by construction conservative, meaning that the total particle strength is time-invariant. The first PSE algorithm estimates the fractional diffusion flux using direct differentiation, and uses an integral representation of the divergence operator. The second one relies on the integral representation of the fractional Laplacian to derive a suitable particle strength exchange formula for the diffusion term. The third PSE construction employs the Green's function of the fractional diffusion equation. A fifth method is developed based on the diffusion-velocity approach, where the diffusion term is transformed into a transport term.

The performance of all five methods is assessed, based on problems for which analytical solutions are known. A detailed analysis is conducted of the various sources of error, namely filtering, quadrature, domain truncation, and time integration. Computational experiments are used to gain insight for the generalization of the proposed constructions, such as applications in bounded domains or variable diffusivity.