A Fully Lagrangian Meshfree Framework for PDEs on Evolving Surfaces

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ABSTRACT

The need to solve PDEs on deforming curves and surfaces arises in various fields, ranging from surfactants, fluid flow, airbag modelling, to biological applications such as biomembranes, and in visualization and image processing. In this talk, we will present a novel framework to solve PDEs on evolving surfaces, with the basis of a moving point cloud.

Lagrangian frames of reference have often been used for volumetric flow applications. Here, we extend them to solve applications on curved manifolds. Most existing methods for solving PDEs on evolving surface require a discretizing mesh, which could need expensive remeshing upon movement of the mesh. Meshfree methods for PDEs on evolving surfaces have been introduced by several authors to avoid this need this need to mesh and remesh. However, they require a discretization not just of the surface, but also one of the bulk volume around the surface. Thus, increasing the dimensionality of the problem. The use of a Lagrangian framework for evolving surfaces in the present talk has the advantage of avoiding this issue. Here, we require only a discretization of the surface, without any need to have a background grid or point cloud of a higher dimension. Just like in the moving surface mesh case, a moving surface point cloud also leads to distortion. However, the key difference is that distortions in the surface point cloud as a result of the movement are fixed by purely local considerations, making it a lot cheaper than remeshing.

We first establish a comprehensive Lagrangian framework for arbitrary movement of curves and surfaces given by point clouds. Collision detection algorithms between point cloud surfaces are introduced, which also allow the handling of evolving manifolds with topological changes. We then couple this Lagrangian framework with a meshfree Generalized Finite Difference Method (GFDM) to approximate surface differential operators, which together gives a method to solve PDEs on evolving manifolds. The applicability of this method is illustrated with a range of numerical examples, which include advection-diffusion equations with large deformations of the surface, curvature dependent geometric motion, and wave equations on evolving surfaces.

REFERENCES

[1] P. Suchde and J. Kuhnert. A fully lagrangian meshfree framework for PDEs on evolving surfaces. *arXiv preprint arXiv:1902.08107*, 2019.