# Local mesh refinement strategies for the Particle Finite Element Method (PFEM)

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## Abstract

The Particle Finite Element Method (PFEM) is a general numerical method that is well adapted to the simulation of free-surface flows and fluid structure interaction problems [1]. The main idea behind it is to combine the advantages of a Lagrangian description for problems with large deformations and the robustness and convergence properties of the classical Finite Element Method. Because the fluid is discretized into Lagrangian particles, one of the key ingredients of the method is the remeshing and boundary detection. This is typically done through a Delaunay triangulation at each time step to ensure a high quality mesh despite large particle displacements. Additionally, the free surfaces / fluid boundaries are identified by applying the alpha shape method [2]. Moreover, some nodes might need to be removed or added to the mesh when the triangulation itself is not able to generate optimal triangular finite elements. The alpha shape technique, and the criteria to add and remove nodes, assume, at least in their basic form, that the size of the mesh elements should be more or less uniform across the domain. However, regions of the flow with large gradients typically require a fine mesh discretization in order to accurately capture the rapid variations of the solution, while a coarser mesh is rather sought in regions where the solution is smooth so as to limit the computational cost.

To combine these conflicting goals, a local scaling based on a local length $L_{\text{loc}}$ is proposed. This enables us to apply all the remeshing criteria in a consistent way. This elementary local length can be based on the typical length of the neighbour elements, on the physics (e.g. by using the amplitude of the velocity gradient), or imposed geometrically (e.g., based on a distance $d$ from a wall). Moreover, the quantitative impact of using high aspect ratio triangular elements as mean to further reduce the computational cost is evaluated.

This local mesh refinement method has been implemented into a 2D PFEM code to solve the incompressible Navier-Stokes equations with a finite element discretization using linear shape functions for both pressure and velocity fields [3, 4]. To avoid the problem of pressure instabilities due to the violation of the Ladyzhenskaya - Babuška - Brezzi (LBB) condition [5, 6], a monolithic pressure stabilizing Petrov-Galerkin (PSPG) method [7, 8] has been used. The new mesh refinement strategy is evaluated in the context of the two-dimensional flow around a circular cylinder at low Reynolds number. In particular, the comparison of the lift, drag, Strouhal number and separation and reattachment points with values of the literature shows that the new approach provides accurate results while reducing the computational cost.

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