

Accurate solution of the boundary integral equation in 2D Lagrangian vortex method at flow simulation around curvilinear airfoils

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ABSTRACT

The problem of numerical solution of the boundary integral equation is considered for 2D case. Viscous vortex domains (VVD) method is used for flow simulation, so vorticity is generated on the whole surface line of the airfoil, and there are a lot of vortex elements close to the airfoil. The aim of the research is to provide high accuracy of numerical solution of the integral equation; at the same time the computational complexity of the numerical algorithm should be at rather low level.

As it is mentioned in [1], it is impossible to develop higher-order numerical scheme without explicit taking into account the curvature of the airfoil surface line. Such schemes, based on the satisfaction of the boundary integral equation with respect to tangent velocity components, are developed in [2]. Further development of the Galerkin approach permits one to develop the third-order of accuracy numerical scheme, based on piecewise-quadratic solution representation on the curvilinear panels. Note, that for such schemes approximate analytic expressions are obtained for the matrix coefficients. This schemes works perfect in the case of potential flow simulation, when vorticity in the flow domain is absent and also when vortex elements are placed rather far from the airfoil surface line.

A successive attempt was made to derive approximate analytical expressions also for the integrals, arising in the right-hand side coefficients for closely placed vortex elements, at least for piecewise-constant and piecewise-linear numerical schemes [2], but such representation of the numerical solution doesn't permit one to reconstruct correctly exact solution in principally if there are vortex elements in the flow domain, placed at the distance smaller than the panel's length to the airfoil surface line. A trivial way to the accuracy improvement which consists in extremely fine surface line discretization, leads to unacceptably high numerical complexity of the numerical algorithm, especially for flow simulation around system of movable airfoils. In order to solve this problem, semi-analytical approach is developed which makes it possible to achieve high accuracy at extremely coarse surface line discretization. It consists of explicit addition of the terms, which correspond to the exact solution taking into account the influence of the vortex elements placed close to the panel, which, in turn, is approximately considered to be arc of a osculating circle.

For example, in the model problem of flow simulation around elliptical airfoil with 2:1 semiaxes ratio only 20 panels are required to achieve the error level less than 1 % (in the most strict L_1 norm for piecewise-continuous distributed solution) for arbitrary position of the vortex element in the flow, while traditional approach requires for this accuracy such surface line discretization, that provides d/h ratio more than 0.75, where d is distance from the airfoil to the vortex element, h is panel size.

That additional terms in the right-hand, which arise in the developed semi-analytical numerical scheme are computed numerically, and the corresponding computational procedure can be efficiently parallelized.

REFERENCES

- [1] G.-H. Cottet and P.D. Koumoutsakos, *Vortex methods: Theory and practice*, CUP (2000).
- [2] I.K. Marchevsky, K.S. Kuzmina and I.A. Soldatova, "Improved Algorithm of Boundary Integral Equation Approximation in 2D Vortex Method for Flow Simulation Around Curvilinear Airfoil", *Mathematics and Mathematical Modeling*, **6**, 22-51 (2018).