

# A Particle Finite Element Approach for the numerical simulation of three-dimensional granular flows

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## ABSTRACT

Dense granular materials are involved in many natural phenomena and in several industrial applications. Flows of granular materials can be found in food, pharmaceutical and chemical sectors where particles are mixed, transported and stored. The capability to predict the motion and the stress distribution of these materials is of paramount importance in the design of industrial equipment. In civil engineering, landslides, debris flows and avalanches are well represented by granular materials. The mathematical description of these materials is complex and, consequently, the use of numerical modeling to predict their motion is difficult.

From the computational point of view, dense granular flows have been numerically modeled with both discrete (e.g. DEM) and continuum methods (e.g. FEM and MPM). This work proposes a continuum based approach in which fluid-dynamics equations are coupled with the so-called “ $\mu(I)$  rheology”, a constitutive law, defined from the experimental evidence on granular flows [1]. If from the one hand this model is quite simple to be implemented in fluid-dynamics code, from the other the equations of motion become highly non-linear and ill-conditioned. This work proposes two regularization approaches which allow the efficient numerical treatment of the “ $\mu(I)$  rheology”.

The Particle Finite Element Method (PFEM) [2, 3] is used for the solution of the governing equations. The original method is here extended to account for the “ $\mu(I)$  rheology” [4]. The proposed formulation has been validated with different tests, in two and three dimensions, comparing the numerical results with experimental data and analytical solutions. The effectiveness of the proposed regularizations has been confirmed by the accuracy of the obtained results.

## REFERENCES

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