Parallel implementation of fast methods for vortex influence computation in vortex methods for 2D incompressible flows simulation

Daria D. Leonova*, Ilia K. Marchevsky*,†, Evgeniya P. Ryatina*†

* Bauman Moscow State Technical University, Russia
† Ivannikov Institute for System Programming of the RAS, Russia

ABSTRACT

Vortex methods are a powerful tool for solving engineering problems of outer incompressible flow simulation around airfoils with small subsonic speeds. They become especially efficient when we deal with arbitrary large body displacements or/and deformations due to absence of the mesh in the flow domain. The main idea is that the vorticity is considered as a primary computed variable. Vorticity distribution is simulated by a set of elementary vorticity carriers (vortex elements). They move in the flow with the velocity, which is superposition of the convective and diffusive ones. In the simplest case the convective velocity of each vortex element can be calculated as the sum of the influences of all the other vortex elements. At every time step of the flow simulation it is necessary to calculate all vortex elements velocities; such problem is similar to the \(N\)-body gravitational problem. Thus the computational complexity of the operation of the vortex elements velocities calculation is proportional to \(n^2\) (\(n\) is number of vortices). This fact restricts significantly the applicability of vortex methods.

Two approximate fast methods having logarithmic \((n \log n)\) computational complexity are implemented and investigated. The first method is an analogue of the Barnes-Hut fast method for the gravitational \(N\)-body problem; the second one is based on the possibility of Poisson’s equation fast solution with respect to the stream function on rather coarse mesh by Fast Fourier Transform (FFT) with further results correction for correct influence accounting of closely-placed vortices. Numerical complexity estimations for both methods are derived, their sequential and parallel implementations are developed. Numerical experiments show that the FFT-based method is more efficient in comparison to the Barnes – Hut method; it provides the acceleration of about 1000 times for the procedure of vortices velocities calculation for the problem with \(n = 500,000\) vortex elements in the flow domain (in comparison to the direct “point-to-point” calculation). The number of mesh cells doesn’t effect the method accuracy, however it determines the computational complexity of the algorithm. So it should be chosen according to used computational resources (multiprocessor systems or/and Nvidia graphic accelerators) for achieving the highest efficiency of the parallelization: there are quite efficient software implementations for the CUDA-based Fast Fourier Transformation, while its MPI implementation has modest efficiency. On the contrary, the correction procedure can be efficiently implemented in parallel mode for MPI multiprocessor systems, but its GPU implementation is not very efficient due to irregularity of vortex elements distribution. However, the FFT-based method is more efficient from the parallelization efficiency point of view in comparison with the Barnes – Hut algorithm due to independence of the calculations for every mesh cell. Due to Barnes – Hut method features, its GPU implementation is very inefficient, but its multi-core parallelization shows quite good acceleration of computations.

REFERENCES