Time Integration Errors and Energy Conservation Properties of the Stormer Verlet Method Applied to MPM

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ABSTRACT

The great practical success of the Material Point Method (MPM) in solving many challenging problems nevertheless raises some open questions regarding the fundamental properties of the method. There has been progress in addressing some of these such as grid crossing and null-space errors and for example nonlinear stability, [2]. The question of the energy conservation of the method has been addressed by Bardenhagen [1] but has not been extended further in a theoretical manner since. In this work both the time integration errors and the energy conservation of MPM is extended from [1] by including the impact of particle movement and grid-crossing and a different time integration approach.

It is shown that the properties of the spatial methods used play an important role in time accuracy and in conservation. In particular it is shown that a lack of smoothness in the spatial basis functions results in a loss of time accuracy. Furthermore it is shown that it is helpful for the basis functions to possess a commutative property. This property is that mapping from particles to nodes followed by differentiation when forming derivatives at the particles is the same as a differentiation mapping to the nodes followed by an interpolation mapping to the particles.

The error in energy conservation is evaluated and as a result, a more accurate method based upon the Stormer-Verlet method [3] is applied. This method is symplectic and very widely used in many applications [4] such as molecular dynamics and planetary orbits and even dates back to Newton as was demonstrated by Feynman [3].

An analysis of this method as applied to MPM is undertaken and the method is shown to have globally second order accuracy in energy conservation and locally third order accuracy of energy conservation in time. This is in contrast to the globally first order accuracy both in the solution and in energy conservation of the Symplectic Euler Methods [4, 2] that are used in many MPM calculations This theoretical accuracy is demonstrated numerically on standard MPM test examples

REFERENCES


