A Cahn-Hilliard Navier-Stokes Method for CFL-Independent Simulations

Niklas Kühl[1*], Thomas Rung[1] and Michael Hinze[2]

[1] Institute for Fluid Dynamics and Ship Theory, Hamburg University of Technology, Am Schwarzenberg-Campus 4, D-21073 Hamburg, E-mail: n.kuehl@tuhh.de

[2] Department of Mathematics, University of Hamburg, Bundesstrasse 55, D-20146 Hamburg

ABSTRACT

This presentation discusses an extension of the classical Volume-of-Fluid (VoF) method with a diffusive term derived from a Cahn-Hilliard (CH) approach [1]. The paper aims at efficient simulations of quasi-static two-phase flows, which can outperform a compressive-scheme based VoF-procedure [2] by almost an order of magnitude computing effort. Formaly, the CH Navier-Stokes system refers to a concentration transport equation which contains a number of additional, unfavourable terms, that are for example associated to 4th-order derivatives or severe non-linearities. Moreover, the VoF-related velocity field is no longer divergence-free. However, if surface tension is insignificant, this equation can be converted into a classical convection-diffusion equation, which inherently provides a sharp phase transition without the use of compressive, downwind-biased approximations of convective kinematics. Introducing an alternative, differentiable equation of state, that is motivated by an analytical solution to two-phase shear-flows, the procedure is strongly regularized. A remaining free mobility constant of the concentration equation is divided into a physical and an artificial (numerical) component. The latter is assigned to computed values that eliminate the numerical diffusion orginating from approximation of the convective kinematics. Using classical upwind formulae, the usual time-step restriction vanishes (Fig. 1). Verification and validation are performed for a variety of 1D and 2D test cases, including analytical solutions. Applictions address 3D simulations of a fully turbulent flow around a container ship at operating points of practical relevance. Examples included demonstrate the robustness and the greatly improved efficiency of the present method.

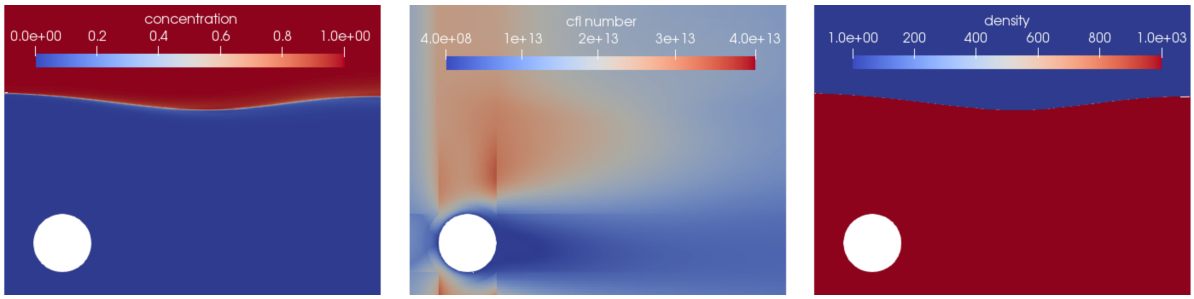

Figure 1: 2D laminar immiscible two phase flow around a towed cylinder at $\mathrm{Re_D} = 20$ and $\mathrm{Fr_H} = 0.5$ approximated with a very large time step. The resulting sharp density field (right) is computed from a diffuse concentration field (left), although the cfl number increased significantly (middle).

REFERENCES

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