Analysing Parameterised Reduced Order Methods

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ABSTRACT

Many systems involve models which depend on parameters. These parameters may either be uncertain quantities, leading to uncertainty quantification (UQ), or they may be design parameters to be used in some kind of design optimisation. In any case, the computational model may have to be evaluated many times for different parameters in the computational process. This leads to the desire to replace models which are costly to evaluate – so-called high-fidelity models (HFM, also called simulator) – with computationally cheaper ones, which give comparable results with maybe some little error. These are typically reduced order models (ROMs), also called surrogate-, meta-, or proxy models, or emulators.

This talk will point out the issues involved in producing such ROMs, and whether the original physics embodied in the original HFM is in some way still preserved in the parametrised ROMs, and which aspects of it are important to be conserved. The original physics in this case may mean several things, it could mean some (weak) satisfaction of the governing equations, or Hamilton´s princple in an adequate form. It could mean preserving important dynamical properties like stability or instability of certain modes. It could also be the property of modelling a couples system, lie e.g. fluid-structure interaction.

Such models may also be seen as following the big data paradigm – from many samples (the big data) – the approximant distills in some way the typical behaviour also for previously not sampled parameter values. Such models may be very good for what they are designed for, but they typically completely leave out the physics embodied in the model. Here we wil show the connections between such big-data models and ROMs.

A completely different approach is the stochastic view. One postulates a ROM of a certain form (which may embody physical principles), but is mainly taken to allow ease of computation or analysis, e.g. a linear model – and then one has to choose some parameters of the ROM in such a way that the output of the ROM and HFM match in a certainy way. This is at this level not too different of what was previously mentioned for physics based models. But now the choice of paramters is performed in a stochastic way, i.e. using some kind of Bayesian approach. This may or may not be constrained by physical considerations.

These different kinds of ROMs, their abilities and properties as well as the computational effort to use or compute them will be investigated, evaluated, and compared. It will be shown that all such ROMs, which may be viewed as a representation of a very general mapping, are intricately connected with a linear map, which is much more amenable to analysis. The decompositions connected to these linear maps naturally lead to tensor-product representations, ant to an analysis of the quality of the ROM.