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ADAPTIVE DISCRETIZATION, REGULARIZATION, LINEARIZATION, AND ALGEBRAIC SOLUTION IN UNSTEADY NONLINEAR PROBLEMS

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We show how computable a posteriori error estimates can be obtained for two model nonlinear unsteady problems, namely the Stefan problem and the multi-phase multi-compositional porous media flow problem. Regularization of the nonlinear functions, iterative linearizations, and iterative solutions of the arising linear systems are typically involved in the numerical approximation procedure. We show how the corresponding error components can be distinguished and estimated separately. A fully adaptive algorithm, with adaptive choices of the regularization parameter, the number of nonlinear and linear solver steps, the time step size, and the computational mesh, is presented. Numerical experiments confirm tight error control and important computational savings.

We present two examples for the multi-phase multi-compositional flow in porous media. In the left part of Figure 1, we plot our estimators of the different error components and of the total error as a function of the GMRes iterations for a fixed time and Newton step. In the right part of Figure 1, we track the dependence with respect to the Newton iterations. Our stopping criteria clearly enable to economize an important number of iterations with respect to the classical ones requiring the relative algebraic/linearization residual to be smaller than 1e-8. The overall gains of our approach are then illustrated in Figure 2, where we plot the cumulated number of necessary Newton and GMRes iterations for both the adaptive and classical stopping criteria. In the adaptive approach, the number of cumulative GMRes iterations is more than 10-times smaller compared to that in the classical one. Details can be found in the references [3] and [2], see also [4] and [1].

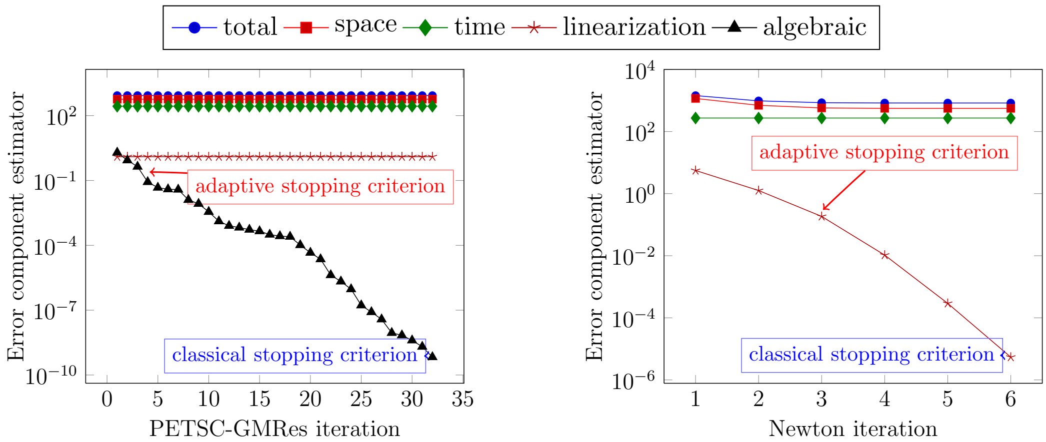

Figure 1: Evolution of the spatial, temporal, linearization, and algebraic error estimators for a fixed mesh and time, as a function of the GMRes iterations on the first Newton iteration (left) and as a function of the Newton iterations (right)

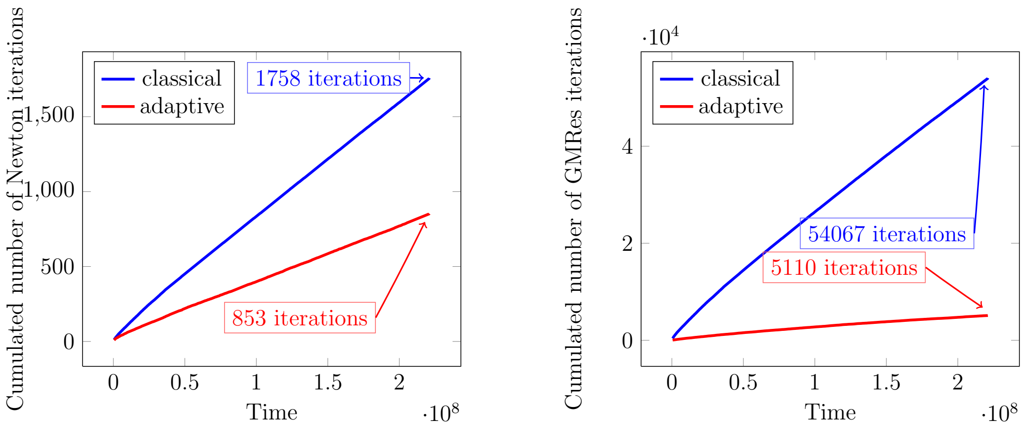

Figure 2: Cumulated number of Newton iterations as a function of time (left) and cumulated number of GMRes iterations as a function of time (right)

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