

ON THE ROBUSTNESS OF A HDG METHOD FOR ELLIPTIC PROBLEMS IN GENERAL DOMAINS

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Key words: *Curved domains, hybridizable discontinuous Galerkin, immerse boundary method*

In this work we present a numerical study of the robustness of the method developed by [1] that approximates the solution of second-order elliptic problems in curved domains. More precisely, we determine conditions that this method must satisfy in order to ensure its robustness with respect to the meshsize h , polynomial degree k and Péclet number Pe .

A technique for solving Dirichlet-boundary value problems in curved domains was introduced in [1] for the pure diffusive case. The domain is approximated by a polygonal subdomain and the boundary condition is transferred to the computational boundary by using suitable defined extension operators. Since the computational domain is polygonal, a hybridizable discontinuous Galerkin method (HDG) is implemented to approximate the solution. Some of the advantages of this technique are that curved domains can be handled, high degree polynomial approximation can be used and there is no need of fitting the mesh to the boundary. In fact, the distance d between the boundary and the computational domain is only of order h . Later, [2] theoretically showed optimal error estimates assuming d of order $h/(k+1)^{8/3}$. However, as the authors in [1] pointed out, in some cases the approximation deteriorates when k increases and the mesh is fixed. This motivated us to study the robustness of the method. In this work we present numerical evidence suggesting that, if d is of order $h/(k+1)^2$, the method is robust with respect to h and k . In addition, for convection-diffusion problems, the method is also robust if d is of order $\min\{h, Pe^{-1}\}/(k+1)^2$.

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