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Why the obstacle reconstruction by topological sensitivity may work

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This work investigates the performance of topological sensitivity as a tool for dealing with the inverse scattering of scalar waves in the high-frequency regime, when the wave length of the incident field is small relative to the remaining length scales in the problem. To provide a focus in the study, it is assumed that the obstacle is convex and impenetrable (of either Dirichlet, Neumann, or Robin type), and that the full-waveform measurements of the scattered field are taken over a sphere whose radius is finite, yet large relative to the size of the sampling region. In this setting, the formula for topological sensitivity is expressed a pair of nested surface integrals – one taken the measurement sphere, and the other over the surface of a hidden obstacle. By way of multipole expansion, the inner integral (over the measurement surface) is reduced to a set of antilinear forms in terms of the Greens function and its gradient. The remaining expression is distilled by evaluating the scattered field on the surface of the obstacle via Kirchhoff approximation, and deploying the method of stationary phase to compute the remaining integral. In this way the topological sensitivity is expressed as a sum of closed-form expressions, signifying the contribution of both isolated and interacting critical points. Thus obtained result explicitly demonstrates the localizing nature of the topological sensitivity and helps better understand some of the reconstruction patterns observed in previous works. On the basis of the analytical developments, a new heuristic is proposed in for the application of topological sensitivity to the high-frequency sensory data. For completeness, the analysis is illustrated by numerical simulations and an application to laboratory observations of the scattered field.