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PATTERN FORMATION FINITE ELEMENT MODELING FOR THIN FILMS ON SOFT SUBSTRATES

Fan Xu[1,2,*], Salim Belouettar[2] and Michel Potier-Ferry[1]

[1] LEM3, UMR CNRS 7239, Université de Lorraine, Ile du Saulcy, 57045 Metz Cedex 01, France. *fan.xu@univ-lorraine.fr, michel.potier-ferry@univ-lorraine.fr*

[2] Centre de Recherche Public Henri Tudor, 29 Avenue John F. Kennedy, L-1855 Luxembourg-Kirchberg, Luxembourg. *fan.xu@tudor.lu, salim.belouettar@tudor.lu*

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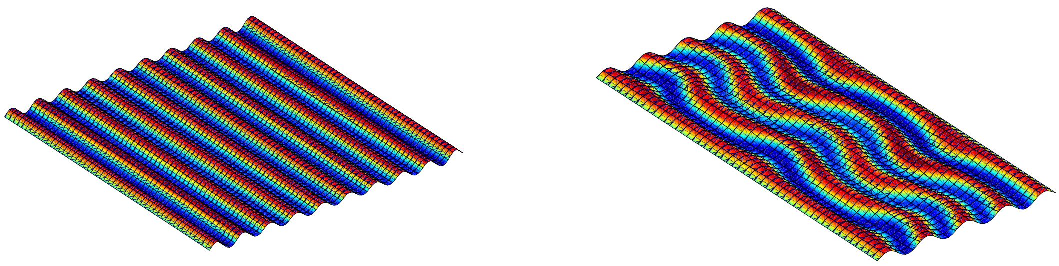
Wrinkles of a stiff thin layer attached on a soft substrate have been widely observed in nature and these phenomena have raised considerable interests over the last decade. In terms of stability study, several theoretical and experimental works have been devoted to linear perturbation analysis and nonlinear buckling analysis. However, most previous studies have been mainly constrained to determine the critical conditions of instability and the corresponding wrinkling patterns near the instability threshold. The post-buckling evolution and mode transition of surface wrinkles are only recently being pursued [1, 2].

This study aims at applying advanced numerical methods for bifurcation analysis to typical film/substrate models and focuses on the post-bifurcation evolution involving secondary bifurcations and advanced modes. For this purpose, a finite element (FE) model based on the Asymptotic Numerical Method (ANM) [3, 4] is developed for multiperiodic bifurcation analysis of wrinkle formation [5]. In this model, the film undergoing moderate deflections is described by Föppl-von Kármán nonlinear elastic theory, while the substrate is considered to be a linear elastic solid. Following the same strategy, we extend the work to 3D situation by coupling shell elements representing the film and block elements describing the substrate. Therefore, large rotations in the film can be considered and the spatial distribution of wrinkling modes like stripe or herringbone can be investigated.

To solve the resulting nonlinear equations, we adopted the ANM which appears as a significantly efficient continuation technique without any corrector iteration. The underlying principle of the ANM is to build up the nonlinear solution branch in the form of relatively high order truncated power series. Since few global stiffness matrix inversions are required (only one per step), the performance in terms of computing time is quite attractive. Moreover, unlike incremental-iterative methods, the arc-length step size in the ANM is fully adaptive since it is determined *a posteriori* by the algorithm. A small radius of convergence and step accumulation appear around the bifurcation and imply its presence. Furthermore, a bifurcation indicator well adapted to the ANM, is computed to detect the exact bifurcation points. This indicator measures the intensity of the system response to perturbation forces. By evaluating this indicator through an equilibrium branch, all the critical points existing on this branch and the associated bifurcation modes can be determined.

Numerical results reveal a sinusoidal mode and herringbone mode under different loading and boundary conditions (see Fig. 1). The evolution of patterns and advanced post-bifurcation mode including period-doubling and localized mode have been observed in the post-buckling range. The results are expected to provide insight into the formation and evolution of wrinkle patterns in film/substrate systems and be helpful to control the surface morphology.



(a) (b)

Figure 1: Wrinkling patterns: (a) sinusoidal mode, (b) herringbone mode.

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