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MULTIRESOLUTION ANALYSIS OF INCOMPRESSIBLE FLOWS INTERACTION WITH FORCED DEFORMABLE BODIES

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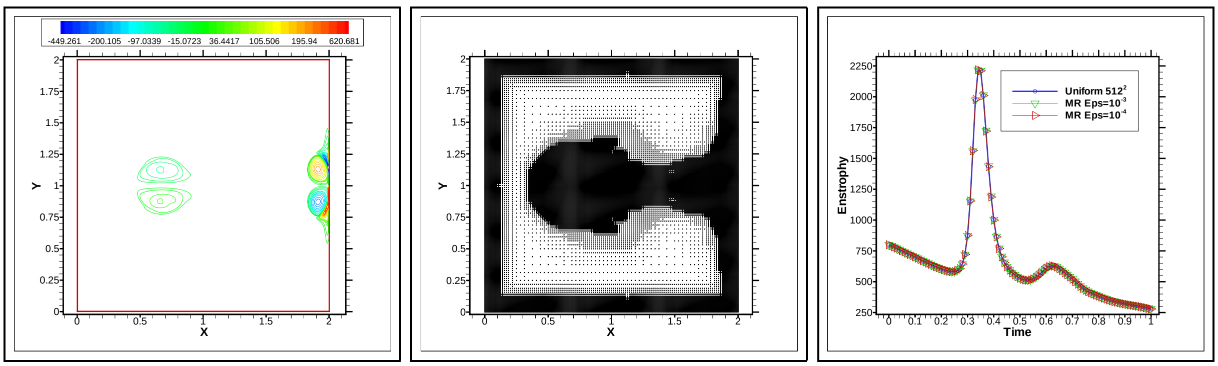
In the present investigation, a space adaptive multiresolution method is developed to solve the incompressible two-dimensional Navier-Stokes equations in the vorticity-stream-function formulations including a forcing term. The penalized vorticity transport equation for two-dimensional flows read

$$\partial_t \omega + (\mathbf{u} \cdot \nabla)\omega = \nu \nabla^2 \omega + \nabla \times [\eta^{-1} \chi (\mathbf{u}_P - \mathbf{u})] \quad , \quad \mathbf{x} \in \Omega \in \mathbb{R}^2 \tag{1}$$

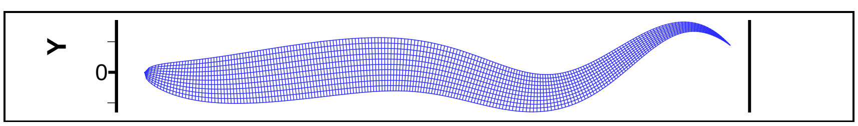
where χ is the mask function covering the penalized zone ($\chi = 1$) in which $\mathbf{u}_P$ is imposed, η is the porosity coefficient, velocity components are $(u, v) = (\partial_y \psi, -\partial_x \psi)$ satisfying; $\nabla^2 \psi = -\omega$. The new method is based on a multiresolution analysis which allows to reduce the number of active grid points significantly by refining the grid automatically in regions of steep gradients, while in regions where the solution is smooth coarse grids are used. To study the accuracy of the method, dipole collision with a straight wall is considered as a benchmark [1], a good agreement between the results of adaptive simulations and that of uniform grid solver is obtained, see Fig. 1. The grid adaptation strategy exploits the local regularity of the solution estimated via the wavelet coefficients at a given time step. Nonlinear thresholding of the wavelet coefficients in a one-to-one correspondence with the grid allows to reduce the number of grid points significantly [1]. Denoting by $E(\Delta t)$ the discrete time evolution operator, the global algorithm can schematically be summarized by

$$\omega^{n+1} = E(\Delta t)\big[M^{-1} \cdot S \cdot T(\epsilon) \cdot M\big]\omega^n \tag{2}$$

where M and M^{-1} are the direct and inverse wavelet transform operators. $T(\epsilon)$ is the thresholding operator, ϵ is the threshold and S represents the safety zone operator. An extension to interactions with forced deformable bodies, i.e., swimming of a fish is done using the volume penalization method. A Lagrangian structure grid with prescribed motion cover the deformable body interacting with surrounding fluid due to hydrodynamic forces and moment calculated on an Eulerian reference Cartesian grid, see Fig. 1-(d). The results of swimming fish are compared with those of [3] where a uniform grid is used. The obtained results show that the CPU time of the adaptive simulations can be significantly reduced with respect to simulations on a regular grid. Nevertheless the accuracy order of the underlying numerical scheme is preserved [2].



(a) (b) (c)



(d) X

Figure 1: Dipole-wall collision. (a) Vorticity contours at $t = 0.35$ for Reynolds 1000. (b) Corresponding adaptive grid points. (c) Comparisons of the total enstrophy $Z(t)$ between the uniform grid solver and the multiresolution computation with thresholds, $\epsilon = 10^{-3}$ and $\epsilon = 10^{-4}$ for maximum grid level $J = 9$ (512) in each direction for all simulations. (d) A Lagrangian deformable structure grid covering the fish.

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