

AN EFFICIENT ALGORITHM FOR SIMULATION OF FORCED DEFORMABLE BODIES INTERACTING WITH INCOMPRESSIBLE FLOWS; APPLICATION TO FISH SWIMMING

Seyed Amin Ghaffari¹, Patrick Bontoux², Stéphane Viazzo³ and Kai Schneider⁴

¹ PhD student, s.amin.ghaffary@gmail.com

² Professor, kscheid@cmi.univ-mrs.fr

³ Professor, stephane.viazzo@L3M.univ-mrs.fr

⁴ Professor, bontoux@L3M.univ-mrs.fr

M2P2-UMR 7340, CNRS, Aix-Marseille Université, Centrale Marseille, France

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We present an efficient algorithm for simulation of deformable bodies interacting with two-dimensional incompressible flows. By taking the curl of the Navier-Stokes equations including penalization term [2], one obtains the penalized vorticity transport equation (1) for two-dimensional flows,

$$\partial_t \omega + (\mathbf{u} \cdot \nabla) \omega = \nu \nabla^2 \omega + \nabla \times [\eta^{-1} \chi (\mathbf{u}_P - \mathbf{u})] \quad (1)$$

where χ is the mask function covering the penalized zone ($\chi = 1$) in which \mathbf{u}_P is imposed, η is the porosity coefficient, velocity components are $(u, v) = (\partial_y \psi, -\partial_x \psi)$ satisfying

$$\nabla^2 \psi = -\omega \quad (2)$$

ψ being the stream function. The time integration of equation (1) is based on classical fourth-order Runge-Kutta method. Spatial discretization is done via fourth-order compact finite differences [1]. By using a uniform Cartesian grid we benefit the advantage of a fourth-order direct solver for the solution of the Poisson equation (2) to ensure the incompressibility constraint down to machine zero. For introducing a deformable body in fluid flow an immersed boundary method is applied to the solution of the Navier-Stokes equations as a forcing term. A Lagrangian structure grid with prescribed motion cover the deformable body interacting with surrounding fluid due to hydrodynamic forces and moment calculated on an Eulerian reference Cartesian grid. Deformation of the backbone is imposed according to

$$y(x, t) = (ax^2 + bx + c) \sin(2\pi(x/\lambda + ft)) \quad (3)$$

See Fig. 1 as an example of penalization method and Fig. 2 for fish swimming in forward gait. The results are compared with those of [3]. Validation of the developed code shows the efficiency and expected accuracy of the algorithm for fish like swimming and also for a variety of fluid/solid interaction problems.

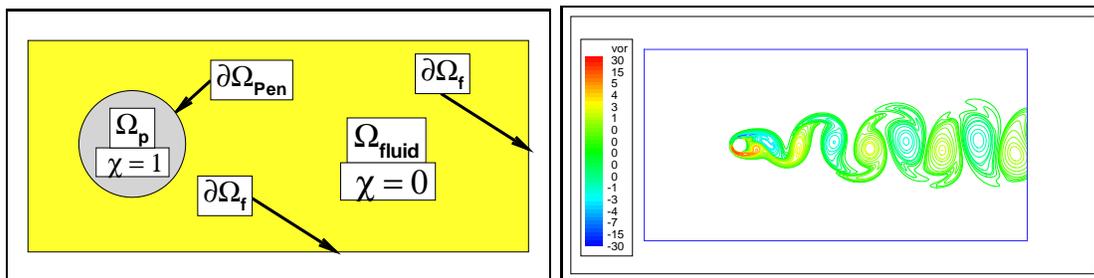


Figure 1: (left) Domain of the solution and the immersed body, $\Omega = \Omega_f \cup \Omega_p$. (right) Vorticity contours for flow around cylinder at $Re = 200$.

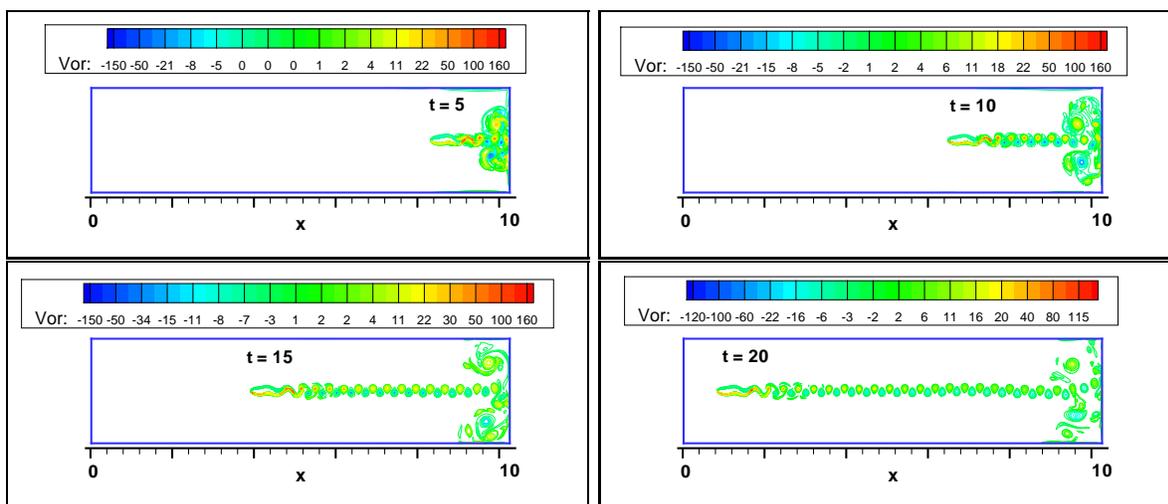


Figure 2: The snapshots of vorticity contours for backbone deformation according to Eq. (3) with $a = 0.16$, $b = -0.08$, $c = 0.2$, $\lambda = 0.5$, $f = 2$, $\eta = 10^{-3}$ and $\nu = 10^{-4}$ for surrounding fluid corresponds to $Re = 4000$.

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