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HP-FEM AND *HP*-DGFEM FOR THE HELMHOLTZ EQUATION

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We consider the question of discretizing the Helmholtz equation at large (real) wavenumber. For high order, piecewise polynomial discretizations we present a convergence theory that is explicit in the discretization parameters (mesh size h and approximation order p) and the wavenumber k. In particular, we show for a class of Helmholtz problems quasi-optimality of the Galerkin discretization in the H^1-norm, if kh/p is sufficiently small and, at the same time, p is at least $O(\log k)$. For a high order discontinuous Galerkin method (DGFEM), we show a similar result under the conditions that $kh/\sqrt{p}$ is sufficiently small and $p = O(\log k)$. While this condition holds for rather general meshes, we show that the condition that $kh/\sqrt{p}$ be small can be relaxed to the condition that kh/p be sufficiently small if the DG-discretization is based on regular meshes.

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