11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

# NATURAL FREQUENCIES OF A SIMPLY SUPPORTED HORIZONTAL RECTANGULAR TANK PARTIALLY FILLED WITH A LIQUID

**Kyeong-Hoon Jeong[1], Jong-Wook Kim[2] and Jong-In Kim[3]**

[1] KAERI, P.O. Box 105, Yuseong, Daejeon, Korea, khjeong@kaeri.re.kr
[2] KAERI, P.O. Box 105, Yuseong, Daejeon, Korea, kjwook@kaeri.re.kr
[3] KAERI, P.O. Box 105, Yuseong, Daejeon, Korea, jikim@kaeri.re.kr

**Key Words:** *Rectangular Tank, Fluid-structure Interaction, Hydroelastic Vibration, Rayleigh-Ritz Method.*

Rectangular tanks in contact with a liquid are the main elements in many types of engineering fields. Although the coupling effect between the tank and the liquid in the liquid-filled vertical rectangular tank was taken into account in previous researches by Zhou et. al [1] and Jeong [2], the dynamic characteristics of a horizontal tank coupled with a liquid was not studied. Hence, a theoretical method to calculate the natural frequencies of a simply supported horizontal rectangular tank partially filled with an ideal liquid is proposed. The theoretical model of the tank with a flexible bottom plate is illustrated in Fig. 1. The tank is assumed as an unfolded rectangular plate with a zero displacement along four corners of the tank. Orthogonal polynomials satisfying the boundary conditions of the tank are extracted using the the Gram-Schmidt process [3], to be used as admissible functions in the Rayleigh-Ritz method. The liquid velocity potential satisfying the Laplace equation and liquid boundary conditions is obtained as a combination of sinusoidal and hyperbolic functions. The compatibility requirement along the contacting surfaces between the tank and liquid is applied for the finite Fourier transform. It leads a relationship between the tank and liquid motions. Finally, an eigenvalue problem is derived so that the wet natural frequencies of the horizontal rectangular tank filled with the liquid can be extracted. The proposed analytical method is verified by observing an excellent agreement with the three-dimensional finite element analysis results using commercial software, ANSYS as shown in Table 1.

## REFERENCES

[1] D. Zhou and W. Liu, Hydroelastic vibrations of flexible rectangular containers partially filled with liquid. *Inter. J. Numer. Methods Engng.* Vol. **71** (2), pp. 149–174, 2007.
[2] K.–H. Jeong, Hydroelastic vibration analysis of liquid-contained rectangular tanks. *Struct. Engng. Mech.* Vol. **40** (5), pp. 665–688, 2011.
[3] P. Cupial, Calculation of the natural frequencies of composite plates by the Rayleigh-Ritz method with orthogonal polynomials. *J. Sound Vibr*., Vol. **201** (3), 385–387, 1997.

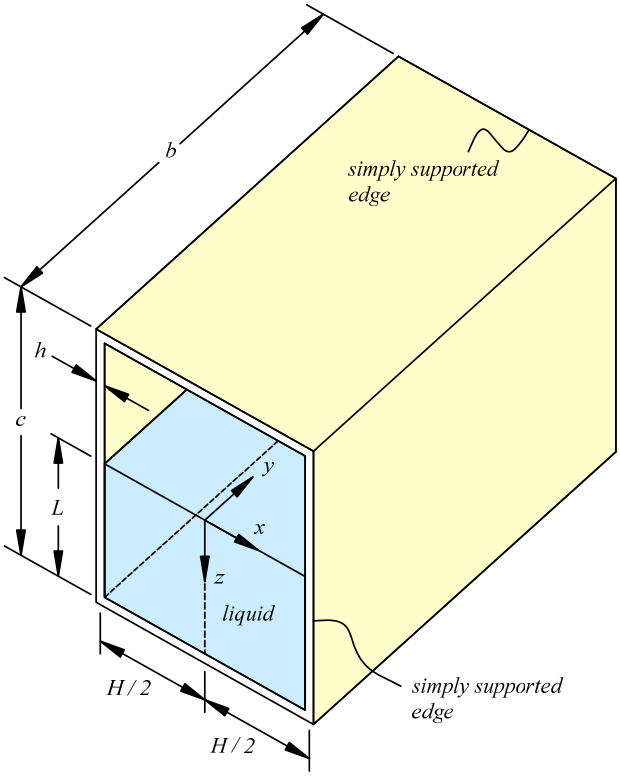


Fig.1: Theoretical model of a rectangular tank partially filled with a liquid

| Serial mode number | Mode shape | | Natural frequency (Hz) | | Discrepancy (%) |
|---|---|---|---|---|---|
| | *x*–*z* plane | *y*–*z* plane | Theory | ANSYS | |
| 1 | S (0,0) | S (0,0) | 38.44 | 38.45 | − 0.03 |
| 2 | S (0,0) | S (0,0) | 52.39 | 51.85 | 1.04 |
| 3 | A (1,0) | S (0,0) | 56.29 | 55.55 | 1.33 |
| 4 | S (0,1) | S (0,1) | 70.69 | 70.34 | 0.50 |
| 5 | S (0,1) | S (0,1) | 95.76 | 94.81 | 1.00 |
| 6 | A (1,1) | S (0,1) | 97.72 | 96.67 | 1.09 |
| 7 | S (0,2) | A (1,2) | 135.27 | 133.27 | 1.50 |
| 8 | S (0,2) | A (1,0) | 140.82 | 140.33 | 0.35 |
| 9 | A (1,0) | S (0,2) | 158.24 | 151.91 | 4.17 |
| 10 | S (0,2) | S (0,2) | 172.80 | 170.93 | − 0.08 |

Table 1: Natural frequencies of the water-filled rectangular tank with a 50% water level.