PARAMETRIZED-BACKGROUND DATA-WEAK FORMULATION FOR STATE AND MODEL BIAS ESTIMATION

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We present an integrated variational framework which combines a best-knowledge mathematical model (parametrized partial differential equation) and data (M experimental observations) to yield estimates for state and parametric and nonparametric model biases. We first abstract the estimation problem as a variational problem in the presence of unlimited observations. We then consider an approximate solution of the variational problem based on experimentally-realizable limited observations. We provide an associated a priori theory which identifies distinct contributions to reduction in the state error with the number of observations. The theory in addition identifies the optimal test and trial space; in the context of our variational data assimilation, the former is associated with the optimal sensor placement and the latter is informed by the best-knowledge parametric manifold. We develop an efficient offline-online computational strategy in the reduced basis setting in which we invoke real data in real-time. We finally apply the method to real data associated with a (three-dimensional) acoustic resonator to assess the effectiveness of the proposed method.