

# A STABILIZED ARBITRARY LAGRANGIAN EULERIAN FINITE ELEMENT METHOD FOR THE MIXED WAVE EQUATION WITH APPLICATION TO DIPHTHONG PRODUCTION

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Though most engineering problems in acoustics directly deal with the wave equation in its irreducible form, in many situations it becomes interesting to consider it in mixed form [1], so as to directly account for both, the acoustic pressure and the acoustic velocity fields. A particular case is that of waves propagating in domains with moving boundaries. When attempting a finite element solution to such problems, the mixed formulation naturally allows to set the equations in an arbitrary Lagrangian-Eulerian (ALE) framework. This results in the appearance of some extra terms involving the scalar product of the mesh velocity and the gradient of the pressure and velocity fields. As known, the finite element solution to the mixed wave equation needs to be stabilized so as to use equal interpolation for the pressure and velocity fields. Following the lines in [1, 2], where algebraic and orthogonal subgrid stabilization [3, 4] were used, in this work a stabilized finite element method is proposed for the ALE wave equation in mixed form.

As an application, we face the problem of the numerical generation of diphthongs. Much numerical work has recently been done with regard to static vocal tract acoustics i.e., generation of vowels and related phenomena (see e.g., [5, 6, 7, 8]), but little has been reported on dynamic vocal tract acoustics, most efforts being placed to date in the simulation of phonation [9]. As a first step towards the generation of diphthongs, some 2D simulations will be presented based on simplified vocal tract geometries [10], which can be tuned to exhibit a 3D behavior [11].

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