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LOCAL HIERARCHICAL P-, HP-, AND K-REFINEMENT IN ISOGEOMETRIC ANALYSIS

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Hierarchical B-spline refinement introduced by Kraft [1] before the advent of Isogeometric analysis, has become a very efficient means of local h-refinement for tensor product structured B-splines. Consequently, it has attracted increasing attention in the Isogeometric analysis community [2, 3, 4, 5].

Hierarchical refinement is based on the subdivision property of B-splines: basis functions on a course grid of characteristic width $2h$ can exactly be represented as a linear combination of basis functions on a finer grid of characteristic width h.

We extend the concept of hierarchical h-refinenement to hierarchical p-refinement: every degree p basis function can be represented exactly as a linear combination of degree $p + 1$ basis functions. Hence, this type of p-refinement is endowed with exactly the same structure as hierarchical h-refinement, and can theredore be easily implemented into existing software that incorporate B-spline hierarchical h-refinement. Furthermore, the non-commutative nature of h- and $p-$ refinement leads to the concept of local k-refinement in Isogeometric analysis.

Our representation of hierarchical B-splines is naturally suited for compatible B-spline discretizations that satisfy a de Rham sequence [6, 7, 8]. We show several 2D-examples of compatible discretization of pde's that result in solutions with shear layers and singularities illustrating the accuracy, simplicity and robustness of local hierarchical p-, hp- and k-refinement.

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