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A COMPUTATIONAL AND ANALYTICAL SOLUTION OF NON-LOCAL STEFAN MELTING PROBLEMS

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The Stefan problem, involving the tracking of the sharp phase front during the melting of a pure solid material, is perhaps the archetypical moving boundary problem. The accessibility to close form solutions makes this problem a natural first stop in verifying the performance of moving boundary numerical schemes. Thus it is more than reasonable to conclude that the numerical solution of Stefan problems offers no serious challenges to the range of currently popular moving boundary solution approaches. There is however, an emerging general interest in understanding systems that exhibit non-local behaviors in space and time. By their nature, a numerical treatment of such problems, requires at each point, storage and calculation of quantities that extend well beyond the current time step and local regions of support; a condition that may impede the application of basic moving boundary solution technologies.

Here we introduce a class of non-local Stefan melting problems, which due to the nature of system heterogeneities, can be expressed in terms of fractional derivatives. We show that the derivation of such formulations arises from the consideration of heat flux terms that can be expressed as weighted sums (convolutions) of thermal gradients through space and time. Models based on both a “diffuse” and “sharp” interface model are derived. We then develop a numerical solution based on the “diffuse interface” model. This approach is verified by comparing against one-dimensional analytical solutions. We observe that the behaviors of the solutions of these fractional non-local Stefan problems differ in two ways from those associated with local models. In the first case, the time exponent of the solution does not take the expected value of $1/2$ (anomalous diffusion). In the second case, in problems with memory (non-locality in time), the solution of the diffuse interface model, as we pass to the sharp interface limit, does not coincide with the sharp interface solution. Our work concludes by providing physical explanations for the observed behaviors and a discussion of open issues towards the development of numerically robust moving boundary solvers.