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A HYBRID UNCERTAINTY QUANTIFICATION METHOD FOR ROBUST OPTIMIZATION

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Nonintrusive sampling methods for uncertainty quantification are quite popular because in this case well developed deterministic computational fluid dynamics (CFD) solvers can be used for flow simulation. The drawback of these methods such as Monte Carlo simulation or stochastic collocation is that they may become highly computationally intensive. Especially for robust optimization with many uncertain input parameters this can be a crucial issue.
In practice, this problem is often avoided by using a deterministic optimization first and afterwards examining the result with an uncertainty quantification in order to check if the result fulfills the given requirements for robustness. However, in this way not necessarily the robust optimum is found.

We propose a combination of an adaptive stochastic collocation method on sparse grids (ASCM) [1] and a sensitivity derivatives based first-order second moment method (FOSM) [2] for uncertainty quantification. Together with a gradient-free Nelder-Mead optimizer [3] and an incompressible CFD solver this forms a hybrid robust optimization scheme which is able to find a robust optimum with less effort than a pure sample method.

The major difference between ASCM and FOSM is in the approximation of the mean $\bar{\mathbf{J}}$ and the variance $\sigma_{\mathbf{J}}^2$ of the objective function $\mathbf{J}$. Both expressions are needed in order to solve the robust optimization problem $\min \mathbf{J}\left(\bar{\mathbf{J}}, \sigma_{\mathbf{J}}^2, \phi, \mathbf{a}\right)$, where ϕ are the state variables and $\mathbf{a}$ the uncertain input variables. The FOSM approximates these terms through

$$\bar{\mathbf{J}} = \mathbf{J}(\bar{\mathbf{a}}) \qquad \sigma_{\mathbf{J}}^2 = \sum_{i=1}^{n} \left(\frac{\partial \mathbf{J}}{\partial a_i} \sigma_{a_i} \right)^2, \tag{1}$$

where $\bar{\mathbf{a}}$ are the mean values of $\mathbf{a}$, σ_{a_i} is the standard deviation of the i^{th} uncertain parameter and n is the number of uncertain parameters. The effort for these calculations depends mainly on the estimation of the derivative and is nearly constant during an optimization run. Since this is only a simple first order method there is no information about the failure which obviously grows with n. In contrast, the ASCM uses an error estimation in order to reach a given accuracy. This is done by the use of more collocation points. This means that during an optimization process the accuracy is nearly constant but the required effort can be higher.

The idea of the proposed hybrid method is that the less computationally intensive but less accurate FOSM is used in the beginning of the optimization for uncertainty quantification in order to get a first rough estimation of the optimum. As soon as the optimizer begins to converge the method for the uncertainty quantification is switched to ASCM and the optimizer starts from this point with adapted parameters to determine a more precise (local) robust optimum. In this way only a few iterations of the expensive ACSM might be needed to find the optimum.

In order to illustrate the advantages and disadvantages of the introduced hybrid robust optimization framework, several numerical examples are presented and discussed. Based on these results it can be seen that the required time to convergence can be reduced as long as the number of uncertain parameters is not too large. Furthermore it is shown that the quality of the uncertainty quantification depends on the chosen type of the derivative computation.

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