

## POTENTIAL BASED MODEL ORDER REDUCTION: THEORY AND GPU IMPLEMENTATION

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Potential based constitutive models such as the concept of generalized standard materials (GSM) are a powerful tool in the modeling of nonlinear solids at small and finite deformation. The potential structure has favorable numerical properties and allows for a variational formulation of the constitutive problem. When it comes to multiscale analysis, the computational cost introduced by classical computational homogenization (e.g., FE<sup>2</sup>) is prohibitively large.

In order to significantly reduce the overall computational complexity novel reduce basis methods incorporating micromechanical properties of the problem setting are used to accelerate the computations. Therefore, an approach based on the Nonuniform Transformation Field Analysis (NTFA, [1]) has been developed by the authors [2]. In this approach, a mixed incremental variation formulation of the microscopic problem is considered at first. Then a reduced basis for the internal variables, i.e. for the plastic strain field and the vector of hardening variables is introduced. This reduced parametrization of the infinite dimensional manifold of internal states captures the geometrical and constitutive properties of the microstructured material. The ansatz functions are referred to as plastic modes and hardening modes, respectively.

Based on micromechanical considerations going back to the TFA of Dvorak and, later, the NTFA, the mechanical balance equations can be solved for any linear combination of these reduced internal variable fields. It remains to determine the evolution of the mode activity coefficients. A key contribution in [?] was the development of a mixed incremental variational multiscale framework in the reduced basis framework which determines the increments of the mode activity from a nonlinear optimization problem. Due to the incorporation of the micromechanics of the problem and by using the specialized ansatz functions determined in a preceding preanalysis stage, the method can provide computational savings in the order of 100 and higher while a good accuracy is preserved.

Besides this methodological speedup, the resulting algorithm has a particular structure that is prone to various algorithmic speedups, e.g., using modern GPU accelerated linear algebraic operations. Recently the authors proposed a massively parallel and highly optimized implementation of the method on Nvidia GPUs [3]. Thereby, an overall speedup of up to  $10^5$  is obtained which, finally, renders three-dimensional multiscale problems feasible for real-world applications. Theoretical and algorithmic aspects of the method are addressed. Illustrative numerical examples are presented and future developments are outlined.

## REFERENCES

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