## MULTIDISCIPLINARY FREE MATERIAL OPTIMIZATION FOR LAMINATED PLATE AND SHELL STRUCTURES

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In Free Material Optimization (FMO) the design variable is the entire elastic material tensor which is allowed vary rather freely over the design domain. The only requirements on the material tensor are that it is symmetric positive semidefinite, with bounded trace to control local stiffnesses. Solution to FMO yields optimal structures by describing the distribution of the amount of material and the local material properties. Therefore, optimal structures found by FMO can be regarded as the ultimately best structures.

The fundamental idea of FMO was introduced in the early 1990s in [1], [2], and [3]. Since then FMO has become one of the growing research areas in structural optimization. In the recent years multidisciplinary FMO problems for two and three dimensional solids have been studied and solved, for example, in [4] and [5]. FMO model for Mindlin plate design is introduced in [1]. Later in [6] FMO formulations are also proposed for plates and shells. However, no FMO models have been proposed for laminate structures, which are nowadays used in many engineering applications. The shell FMO models in [6] are analogous to the recent FMO formulations for solid structures. We extend these formulations and propose new FMO models for laminated plate and shell structures, see Figure 1. One cause for structural failure is high stresses. We include constraints on local stresses in the proposed formulation to control them.

FMO problems belong to a class of non convex semidefinite programing (SDP), a nonstandard problem with many matrix inequalities that needs special optimization methods. We propose a primal-dual interior point method for FMO that efficiently solves large-scale FMO problems. The method is developed by extending existing primal-dual interior point methods for nonlinear programming and coupling it with methods for linear SDP. The method and the implementation exploit the special structure that FMO problems have many but small matrix inequalities.

We solve large-scale FMO problems for laminated plate and shell structures with stress

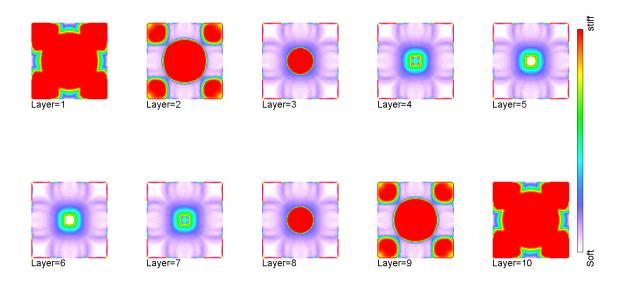


Figure 1: Optimal density distribution of a simply supported laminated plate structure of ten layers under single transverse load concentrated at the center.

constraints. The method requires a modest number of iterations which marginally increases with problem size. It also obtains a higher quality solutions compared to solutions obtained by other methods precisely developed for FMO for solid and shell structures.

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