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CONTACT ANALYSIS IN THE PRESENCE OF AN ELLIPSOIDAL INHOMOGENEITY WITHIN A VISCOELASTIC HALF SPACE

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A semi-analytical three-dimensional contact model for heterogeneous viscoelastic materials is presented. The formalism is based on the work of Jacq et al. [1], with an anisotropic ellipsoidal inhomogeneity embedded in a viscoelastic half space. The viscoelastic property of the half-space is supposed to be linear. The generalized Maxwell (Figure 1) model [2] is used to describe the viscoelastic behavior of the half-space.

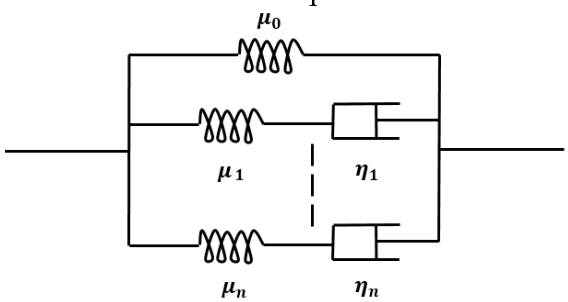

Figure 1: Generalized Maxwell model

The relaxation modulus can be written as:

$$R(t) = \left[\mu_0 + \sum_{i=1}^{n} \mu_i \exp\left(-\frac{t}{\mu_i}\eta_i\right)\right] H(t) \tag{1}$$

A normal contact problem between two bodies can be described by a set of equations that must be solved simultaneously. These equations are:

- The load balance. The integration of contact pressure $p(x_1, x_2, t)$ must be equal to the applied external load $W(t)$.

$$W(t) = \iint_{\Gamma_C} p(x_1, x_2, t)\, dx_1 dx_2 \tag{2}$$

- The surface separation. The distance between the contacting surfaces $h(x_1, x_2, t)$ is defined by the initial geometry $h_i(x_1, x_2)$, the rigid body displacement δ and by the sum of the normal elastic displacements of the contacting surfaces $u_3(x_1, x_2, t)$:

$$h(x_1, x_2, t) = h_i(x_1, x_2) - \delta + u_3(x_1, x_2, t) = 0 \tag{3}$$

The surface normal displacement, $u_3(x_1, x_2, t)$, at time t of a viscoelastic surface can be expressed as,

$$u_3(x_1,x_2,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{t} G(x_1 - x_1', x_2 - x_2', t - \xi)\frac{\partial p(x_1', x_2', \xi)}{\partial \xi}\, dx_1' dx_2' d\xi \qquad (4)$$

where, $G(x_1,x_2,t)$ is the viscoelastic Green's function.

- The contact conditions. The bodies cannot interpenetrate one another; thus $h(x_1,x_2,t)$ must be positive or nil. If $h(x_1,x_2,t)$ is not nil, no contact occurs and no pressure is transmitted:

$$\begin{array}{ll} h(x_1,x_2,t) = 0 & \text{and } p(x_1,x_2,t) > 0 \\ h(x_1,x_2,t) > 0 & \text{then } p(x_1,x_2,t) = 0 \end{array} \qquad (5)$$

Equations (3-5) are discretized in time and in spatial dimensions. The equivalent inclusion method (EIM) of Eshelby [3, 4] is modified in order to take into account the effect of the ellipsoidal inhomogeneity in the contact solver [5]. 3D and 2D Fast Fourier Transforms are used to improve the computational efficiency. The computing time and allocated memory are kept small, compared to the finite element method, by the use of elementary analytical solutions implemented in the numerical algorithm. Figure 2 presents the pressure distribution obtained for an ellipsoidal inclusion within an isotropic matrix.

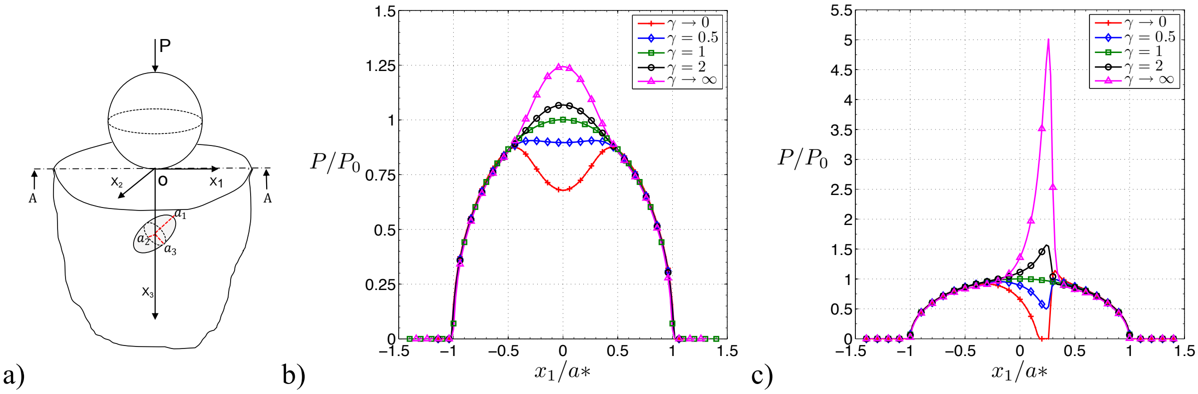

Figure 2: Effect of the dimensionless Young's modulus $\gamma = E_I/E_M$ (with $\nu_I = \nu_M = 0.3$) on the dimensionless contact pressure distribution for isotropic ellipsoidal inclusion ($a_1 = 0.4a$, $a_2 = a_3 = 0.1a$, $z = 0.3a$); (a) geometry, (b) θ=0 and (b) θ=45∘.

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