11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

OPTIMAL PRECONDITIONING FOR THE COUPLING OF ADAPTIVE FINITE AND BOUNDARY ELEMENTS

Michael Feischl[1], Thomas Führer[1,*], Dirk Praetorius[1] and Ernst P. Stephan[2]

[1] Institute for Analysis and Scientific Computing, Vienna University of Technology,
Wiedner Hauptstrasse 8–10, A–1040 Vienna,
{michael.feischl,thomas.fuehrer,dirk.praetorius}@tuwien.ac.at
[2] Institute for Applied Mathematics, Leibniz University Hannover,
Welfengarten 1, D–30167 Hannover,
stephan@ifam.uni-hannover.de

Key words: *FEM-BEM coupling, preconditioner, multilevel additive Schwarz, adaptivity.*

For many relevant applications, the coupling of the finite element method (FEM) and boundary element method (BEM) appears to be the appropriate numerical method to cope with unbounded domains. As the problem size increases, so does the strong need for effective preconditioners for iterative solvers. Most of the available literature on preconditioning of FEM-BEM coupling techniques deals with the symmetric coupling on quasi-uniform meshes, e.g. [3]. In practice, however, non-symmetric coupling formulations are preferred, since they avoid the computation and evaluation of the hypersingular integral operator. The non-symmetric Johnson-Nédélec coupling is considered in [4], but the analysis of the preconditioner relies on compactness of the double-layer potential. First, this restricts the applicability and excludes, e.g., Lamé-type transmission problems. Second, the compactness of the double-layer potential requires smooth boundaries and thus prohibits corners and edges in the problem geometry.

We present own results [2] on block-diagonal preconditioning for the Johnson-Nédélec coupling on adaptively generated meshes. With an appropriate stabilization vector $\mathbf{S}$, which ensures positive defineteness of the coupling formulation [1], the Galerkin matrix of the Johnson-Nédélec coupling reads in block form

$$\begin{pmatrix} \mathbf{A} & -\mathbf{M}^T \\ \frac{1}{2}\mathbf{M} - \mathbf{K} & \mathbf{V} \end{pmatrix} + \mathbf{S}\mathbf{S}^T.$$

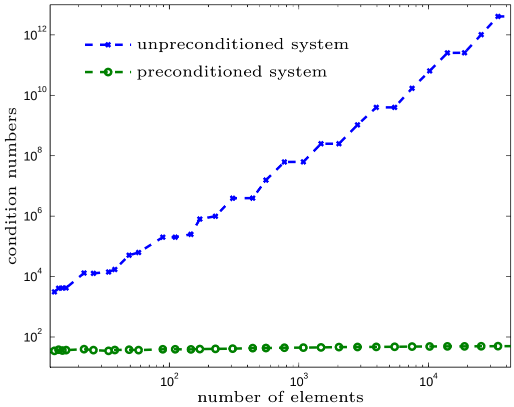
The matrix block $\mathbf{A}$ is the (positive semi-definite) Galerkin matrix of the FEM part, and $\mathbf{V}$ is the Galerkin matrix for the discrete simple-layer integral operator. The matrix block $\mathbf{M}$ is the mass matrix, and $\mathbf{K}$ corresponds to the discrete double-layer integral operator.

As in [5], we consider block-diagonal preconditioners

$$\begin{pmatrix} \mathbf{P}_{\text{FEM}} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{\text{BEM}} \end{pmatrix},$$

where the diagonal blocks $\mathbf{P}_{\text{FEM}}$ and $\mathbf{P}_{\text{BEM}}$ are symmetric and positive definite. These are obtained from a local multilevel additive Schwarz decomposition of the energy space. While the analysis relies on this abstract frame, the resulting preconditioners are obtained from simple algebraic postprocessing of the (history of the) Galerkin matrix.

Starting from an initial mesh which is adaptively refined by bisection, we prove that the condition number of the preconditioned system remains bounded, where the bound depends only on the initial mesh. Moreover, we prove that the maximal number of preconditioned GMRES iterations which are needed to satisfy a relative tolerance of the algebraic GMRES residual remains bounded as well. In conclusion, this proves that the proposed block-diagonal preconditioner is optimal.



Although we shall mainly discuss the 2D Laplace transmission problem, the principal ideas also apply to the 3D case and Lamé-type transmission problems. Moreover, the analysis transfers to other coupling methods, such as the symmetric coupling or the symmetric and non-symmetric Bielak-MacCamy coupling.

REFERENCES

[1] M. Aurada, M. Feischl, T. Führer, M. Karkulik, J.M. Melenk and D. Praetorius. Classical FEM-BEM coupling methods: nonlinearities, well-posedness, and adaptivity. *Comput. Mech.*, Vol. **51**, 399–419, 2013.

[2] M. Feischl, T. Führer, D. Praetorius and E.P. Stephan. Optimal preconditioning for the symmetric and non-symmetric coupling of adaptive finite elements and boundary elements. *ASC Report 36/2013*, 2013.

[3] N. Heuer, M. Maischak, E.P. Stephan. Preconditioned minimum residual iteration for the h-p version of the coupled FEM/BEM with quasi-uniform meshes. *Numer. Linear Algebra Appl.*, Vol. **6**, 435–456, 1999.

[4] S. Meddahi. An optimal iterative process for the Johnson-Nédélec method of coupling boundary and finite elements. *SIAM J. Numer. Anal.*, Vol. **35**, 1393–1415, 1998.

[5] P. Mund and E.P. Stephan. The preconditioned GMRES method for systems of coupled FEM-BEM equations. *Adv. Comput. Math.*, Vol. **9**, 131–144, 1998.