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SHARP ESTIMATES FOR SOME PROBLEMS WITH FADING MEMORY

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Now that computational science and engineering has become established alongside theory and experiment as the 'third leg' of innovation, the development of robust software that can quantify modelling and discretization error is of utmost importance. A powerful paradigm for this quantification connects 'error' to 'computable data' through the solution of a dual problem. While the *dual weighted residual* variant of this approach can provide very high quality adaptive strategies, the need to solve the dual problem and store its solution renders its use prohibitive (in many cases) for space-time problems. The alternative is to eliminate the dual solution using data-stability estimates of the form $\|\text{solution}(t)\|_U \leqslant S(t)\|\text{data}\|_D$ for $S(t) \sim e^t$. The exponential growth is predicted by **Gronwall's lemma** and has a profound work-increasing effect on error control since the resulting *a posteriori* bound will have $S(t)$ on the right hand side.

This presentation will summarise a collection of results and techniques that **avoid Gronwall's lemma** and thus provide sharper stability factors, $S(t)$, for **partial differential Volterra equations** (PDVE's) with fading convolution memory such as,

$$\partial_t^n u(t) + Au(t) = f(t) + \int_0^t B(t-s)u(s)\,ds.$$

Here ∂_t^n denotes n-th order partial time differentiation, A is an elliptic operator with $B(t)$ 'similar' to A and decaying in time in ways that will be explained. (For example: $A = -\nabla^2$ and $B(t) = Ae^{-t}$ or $B(t) = At^{-1/2}$.) Such models arise in various application areas related to dispersive or 'lossy' materials. For example, in viscoelasticity (polymers, soft tissue, wood, concrete, ...) we have the Lamé operator $A = (\lambda + \mu)\nabla\nabla\cdot + \mu\nabla^2$ whereas for the electromagnetism of Debye or Lorentz media we have $A = \nabla\times\nabla\times$. In both cases $n = 2$ although the quasistatic assumption ($n = 0$) is useful in stress analysis. The case $n = 1$ arises in the study of heat conduction and diffusion in viscoelastic media.

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