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UNCERTAINTY QUANTIFICATION IN FLUID DYNAMICS: KRIGING MODEL BASED APPROACH

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This paper proposes a new Kriging-surrogate-model-based non-intrusive uncertainty quantification (UQ) method for an accurate and effective UQ in computational fluid dynamics. The Kriging surrogate model [1] can adapt well to non-linear functions, and estimates not only the function values $\hat{f}(\boldsymbol{\xi})$ but also their fit uncertainties $\hat{s}(\boldsymbol{\xi})$ that are equivalent to approximated errors as shown in Fig. 1.

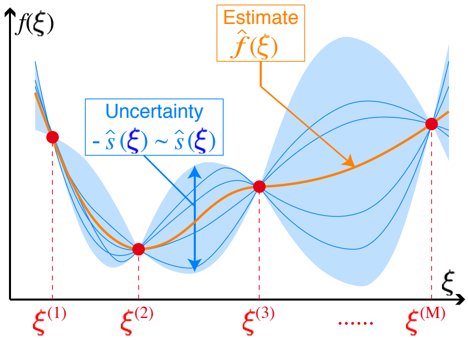

Figure 1: Kriging model.

The first argument of the proposed approach is that since the Kriging model adapt well to non-linear functions we propose to use the Kriging model for approximating stochastic behaviors of output solution that can be discontinuous due to the strong non-linearity of fluid equations. The second argument is to use both the fit uncertainty and the gradient information of the Kriging predictors for dynamic adaptive sampling. By introducing criterion $\mathrm{Crit}(\boldsymbol{\xi})$, a new sample of output solution $f(\boldsymbol{\xi})$ is added at the location $\boldsymbol{\xi}$ in the stochastic space where the value of criterion $\mathrm{Crit}(\boldsymbol{\xi})$ is maximized, and here we propose effective criteria for dynamic adaptive sampling as

$$\mathrm{Crit}(\boldsymbol{\xi}) = \left(|\Delta\hat{f}(\boldsymbol{\xi})| + D_{\hat{f}}(\boldsymbol{\xi})\right) \times \hat{s}(\boldsymbol{\xi}) \times \mathrm{PDF}(\boldsymbol{\xi}). \tag{1}$$

$\Delta\hat{f}(\boldsymbol{\xi})$ is the n-dimensional vector of the finite difference of $f(\boldsymbol{\xi})$ with respect to $\boldsymbol{\xi}$ whose k-th element is defined as $\left[\Delta\hat{f}(\boldsymbol{\xi})\right]_k = \hat{f}(\boldsymbol{\xi}+\boldsymbol{e}_k\Delta\xi) - \hat{f}(\boldsymbol{\xi}-\boldsymbol{e}_k\Delta\xi)$. $\boldsymbol{e}_k$ is an n-dimensional unit vector, and the spacing $\Delta\xi$ is set to be the distance from $\boldsymbol{\xi}$ to the most adjacent sample point, i.e., $\Delta\xi = \min_{i=1,2,\cdots,N} |\boldsymbol{\xi} - \boldsymbol{\xi}^{(i)}|$. An extra term $D_{\hat{f}}(\boldsymbol{\xi})$ in Eq. 1 is expected to estimate the errors in response surfaces. This paper formulates $D_{\hat{f}}$ as $D_{\hat{f}}(\boldsymbol{\xi}) = |\hat{f}(\boldsymbol{\xi}) - \hat{f}_{\mathrm{pre}}(\boldsymbol{\xi})|$. $\hat{f}(\boldsymbol{\xi})$ is predicted from the current sample set (*e.g.*, N points), and $\hat{f}_{\mathrm{pre}}(\boldsymbol{\xi})$ is predicted from the previous sample set ($N-1$ points).

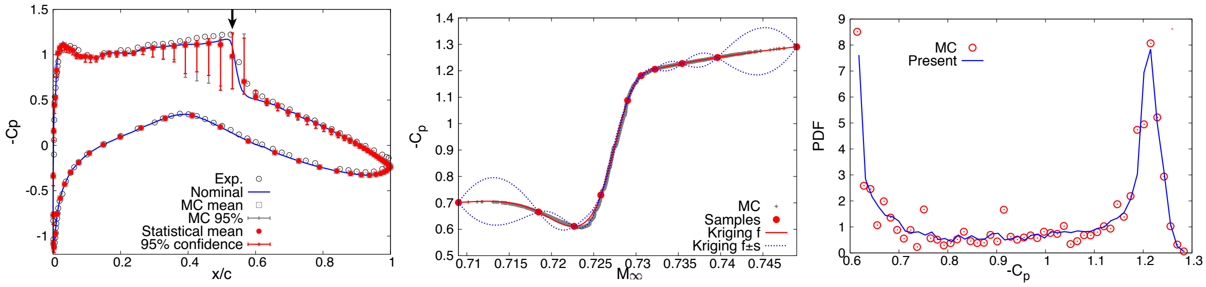

Figure 3: Pressure distributions around the airfoil with statistical mean and 95% confidence intervals (left), C_p at $x/c = 0.53$ (insert arrow location) in the stochastic space (center), and its probability density function (right).

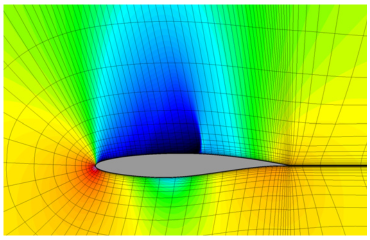

Figure 2: Transonic RAE 2822 airfoil flow.

The proposed Kriging-based UQ method is first tested on analytic functions with smooth and non-smooth response surfaces, and then applied to the transonic RAE 2822 airfoil flow under uncertainties in freestream Mach number by combining the Kriging-model-based UQ method with CFD (RANS simulation). Here we show the results of transonic flow around RAE 2822 airfoil (Fig. 2) at nominal freestream Mach number $M_{\infty,\mathrm{Nominal}} = 0.729$, angle of attack $\alpha = 2.31$, and Reynolds number $Re_c = 6.5 \times 10^6$ under the normal uncertainty in freestream Mach number with the standard deviation of $\sigma_M = 0.005$. The results obtained by the present Kriging-based UQ method with 10 sample simulations are compared with the results obtained by Monte Carlo method with 10,000 samples in Fig. 3. Only with 10 sample data, the present Kriging-based UQ method yields accurate approximation of the non-smooth response and its probability density function where the conventional polynomial chaos expansion (PCE) method often suffers from errors across the non-smooth response. The results also suggest that the output uncertainty does not follow the same probability density function (PDF) as the input uncertainty because of the non-linearity of fluid equations, and the pressure takes two distinct values either high or low pressures in PDF because the shock moves forth and backward depending on a slight change in Mach number. Although not shown here, the aerodynamic coefficients (i.e., lift, drag and pitching moment coefficients) show linear smooth stochastic behaviors and the present UQ method accurately predicts their behaviors with even fewer 5 sample data.

The present results demonstrated that the Kriging surrogate model is a very robust surrogate method to approximate non-linear stochastic behaviors for uncertainty quantification in fluid dynamics. Also the proposed dynamic adaptive sampling method yields accurate approximation of both smooth and non-smooth responses and their probability density function with the order of 10 samples, which is a superior performance compared to the existing PCE method.

REFERENCES

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