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Nonlocal diffuse interface models for two-phase fluids

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The diffuse interface approach is the natural extension of van Der Waals' theory of critical phenomena. In particular, it is also used to give a more tractable description of problems that are characterized by the presence of sharp interfaces (e.g., solid-liquid phase transitions, mixtures of fluids, liquid crystals, superconductive materials). The method essentially consists in replacing the sharp interface with the level set (the so-called diffused interface) of a smooth function called order parameter (or phase-field) whose evolution is governed by a suitable nonlinear PDE. Such an equation depends on the thickness of the diffuse interface. It is usually possible to show (formally, at least) that the sharp interface problem can be obtained by letting the thickness go to zero. The phase-field approach is very effective from the numerical viewpoint (e.g., no interface tracking) and yields a satisfactory description of the underlying phenomenon. A typical example is the so-called Caginalp system, that is, a semilinear parabolic equation for the phase-field coupled with the temperature equation. This is a model for solid-liquid phase transitions whose sharp interface limit is the Stefan problem. Another interesting class of phase-field models arises in multiphase flows. For instance, the evolution of a mixture of two incompressible and partially miscible viscous fluids can be modeled through the so-called Cahn-Hilliard-Navier-Stokes (CHNS) system. This model consists of the Navier-Stokes equations which are nonlinearly coupled with a convective Cahn-Hilliard equation through the so-called Korteweg force.

The CHNS system has been analyzed by several authors both from the numerical and theoretical viewpoints. Here I intend to give first an overview of some recent theoretical results on CHNS systems characterized by a nonlocal Cahn-Hilliard equation (see, e.g., [1-3] below). Such an equation seems physically more appropriate than its approximation, that is, the classical fourth-order Cahn-Hilliard equation.

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