11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

EXPLORING FRAME STRUCTURES WITH NEGATIVE POISSON'S RATIO VIA MIXED INTEGER PROGRAMMING

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Key words: *Negative Poission's ratio, Auxetic structure, Topology optimization, Integer optimization, Local search.*

Materials with negative Poisson's ratio will expand transversely when stretched longitudinally. Although most of materials are characterized by Poisson's ratio in the range $0 < \nu < 1/2$, some naturally occurred materials, including cadmium and single crystals of arsenic, exhibit negative Poisson's ratio. Such a material is called an *auxetic material*. Extensive study of artificial auxetic materials was initiated by development of auxetic polymer foams due to Lakes [1]. A methodology of auxetic material design using structural optimization was initiated by Sigmund [2], in which a repetitive base cell is modeled as a truss structure. Continuum topology optimization based on a homogenization method was also applied to this problem [3].

This paper addresses design of planar periodic frame structures exhibiting auxetic properties. We formulate a topology optimization problem that minimizes the ratio of the output displacement to the prescribed input displacement of a frame structure. Section of each beam element is chosen from a predetermined set of some candidates. That is, the cross-sectional area and moment of inertia of member i, denoted by a_i and I_i, satisfy

$$(a_i, I_i) \in \{(0,0)\} \cup \{(\bar{a}_p, \bar{I}_p) \mid p \in P\}. \tag{1}$$

This constraint, as well as the other constraints considered in this paper, can be treated within the framework of *mixed integer linear programming* (MILP). Let $x_{ip} \in \{0,1\}$ $(\forall p \in P)$ be variables satisfying

$$\sum_{p \in P} x_{ip} \leq 1.$$

Then constraint (1) can be rewritten as

$$a_i = \sum_{p \in P} \bar{a}_p x_{ip}, \quad I_i = \sum_{p \in P} \bar{I}_p x_{ip}.$$

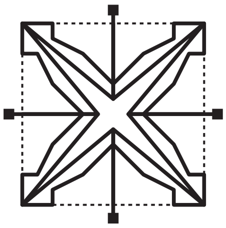
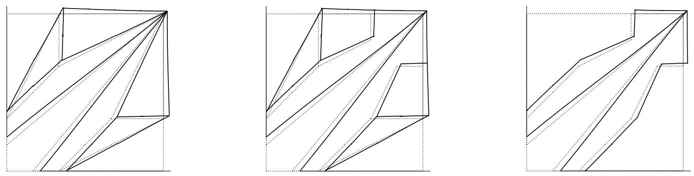


Figure 1: Obtained base cell.



(a) (b) (c)

Figure 2: Convergence history of the local search. The solutions obtained at (a) the 1st iteration; (b) the 2nd iteration; and (c) the 3rd iteration.

Local stress constraints, i.e., bounds for axial force and two end moments, are also imposed on existing beam element. Therefore, the optimal solution has neither hinges nor thin members.

An MILP problem can be solved globally by using, e.g., a branch-and-cut method. Commercial software packages are available for this purpose. This approach, however, can be applied to relatively small problems from a viewpoint of computational cost. Larger problems are attacked by the following local search. We begin with a ground structure consisting of a small number of members that can be solved globally by the MILP approach. Next we prepare a finer ground structure by dividing a member into two members with adding new nodes. Then the optimal solution for the coarse ground structure is translated to the finer ground structure. This solution used as an initial point for the local search, that is performed in the neighborhood defined by

$$\sum_{i\in E}\sum_{p\in P}|x_{ip}-x_{ip}^{*}| \leq r.$$

Here, x_{ip}^{*} is the initial point and $r > 0$ is the specified radius of the neighborhood. This local search is essentially same as what proposed by Stolpe and Stidsen [4].

In numerical experiments it is demonstrated that periodic frame structures exhibiting negative Poisson's ratio properties can be obtained by the proposed method. Figure 1 shows an example of the obtained base cell, which has $\nu = -0.969188$. Figure 2 shows convergence history of the local search, where only a quarter of the base cell is illustrated. Thus the solutions obtained by the proposed method do not involve hinges because they are frame structures. This may be regarded as an advantage in manufacturability in the sense that no post-processing is required.

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