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A couple stress theory for the analysis of plates with a RBF-FD meshless method

Carla M. C. Roque*[1], António J. M. Ferreira[2]

[1] IDMEC-Polo FEUP, Universidade do Porto
Rua Dr. Roberto Frias, 404, 4200-465 Porto, Portugal,
croque@fe.up.pt

[2] Universidade do Porto,
Rua Dr. Roberto Frias 404, 4200-465 Porto, Portugal,
ferreira@fe.up.pt

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Experimental observations indicate that the mechanical behavior of micro and nano systems cannot be accurately simulated by using classical deformations theories. Analytical and numerical studies of nano and micro structures can be developed using atomic and molecular models or continuum models. Atomic models, although adequate for the nano scale, are complex and computational expensive whereas continuum models provide a much simpler approach.

The modified couple stress theory has been mostly applied to beams and was previously used for the study of free vibrations analysis of micro plates and static deflection of Kirchhoff and Mindlin plates [1, 2]. The strain energy for an isotropic linearly elastic material in domain Ω is:

$$U = \int_{\Omega} \frac{1}{2}(\sigma : \epsilon + m : \chi) dV$$

where dV is the volume element, σ is the symmetric part of the Cauchy stress tensor, m is the deviatoric part of the couple stress tensor and χ is the symmetric curvature tensor.

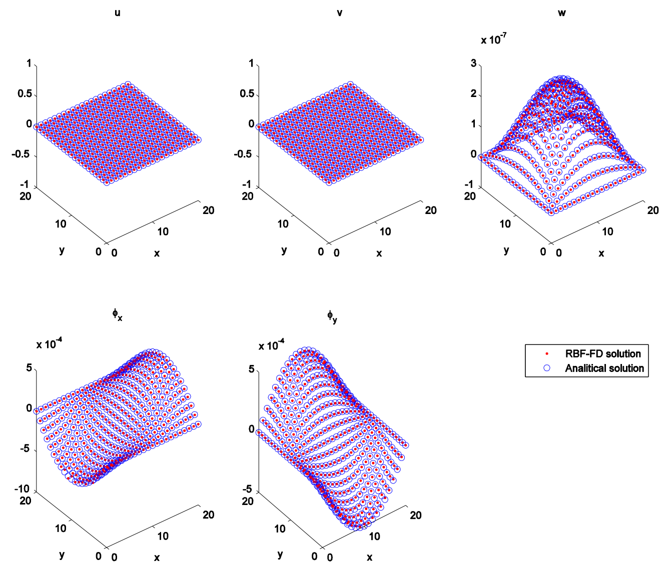
In the present paper, a modified couple stress theory [2] and a meshless method is used to determine the static bending solution of simply supported isotropic micro plate according to the first-order shear deformation plate theory. The meshless method with collocation is used to solve the resulting boundary value problem and the results are compared with Navier analytical solutions.

Radial basis functions were used by Hardy for the interpolation of geographical scattered data and later used by Kansa for the solution of partial differential equations (PDEs), with a global collocation. The global collocation proposed by Kansa considers a set of points distributed over a domain and boundary of the problem. Each point is connected, through a radial basis function, to all the remaining points of the nodal set. This global collocation generally produces dense, unsymmetrical, ill-conditioned matrices, which in turn can produce poor results and instability in the solution. However, when properly used, the global collocation produces excellent results.

As a result of the ill-conditioning, some authors proposed a radial basis function method with a local approach [3]. The idea is to use radial basis functions with a local collocation as in finite differences, reducing the number of connections (the so-called support) for each node (also called center), hence producing a sparse matrix. This local approach retains many of the advantages of the global collocation, yet reducing the conditioning of the matrix.

Numerical Example

A simply supported square plate of length a and thickness $h = 17.6 \times 10^{-6}$ under uniform load is considered. The ratio length/thickness is constant $a/h = 20$. The value for shear correction factor Ks is taken to be 0.8. Material properties are E = 1.44×10^9 and $\mu = 0.38$. Numerical results for deflection are normalized as $w = w/h$; $a = x/h$. Results are compared with an analytical solution. Figure 1 shows the numerical and analytical solutions for displacement filed variables u, v, w, φ_x and φ_y .



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