

# TRANSONIC NONLINEAR AEROELASTIC SIMULATIONS USING AN HARMONIC BALANCE METHOD

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This paper presents an approach to compute transonic Limit Cycle Oscillations using a coupled Harmonic Balance formulation based on the Euler equations for fluid dynamics and finite element models. The paper will investigate the role of aerodynamic (shocks) and structural nonlinearities in driving the limit cycle behaviour. Particular attention will be given to nonlinear interactions for subcritical LCOs. The Aeroelastic Harmonic Balance formulation represented in eq.(1), allows for solutions of the coupled structural dynamics and CFD system at a reduced cost. The Harmonic Balance formulation of the coupled CFD-CSD system can be expressed as:

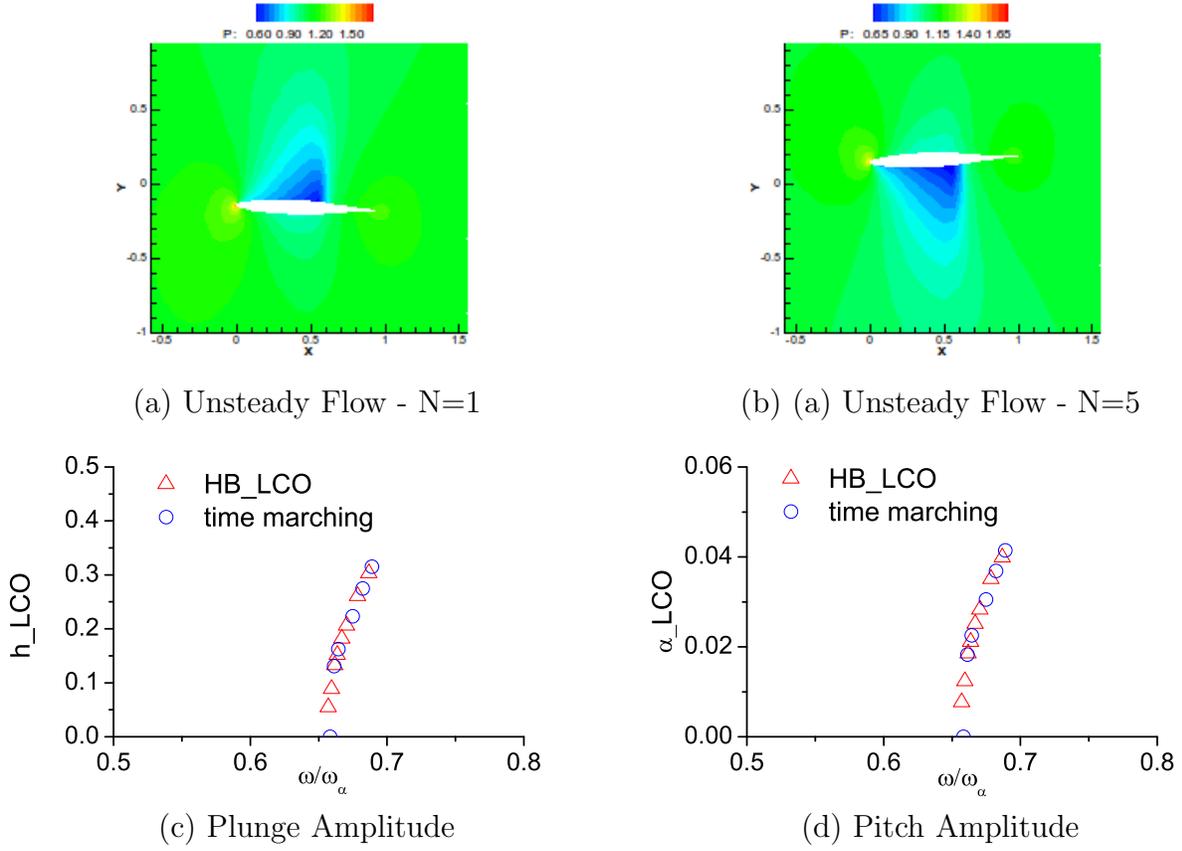
$$\frac{d\mathbf{W}_{hb}}{d\tau} + \omega \mathbf{D}\mathbf{W}_{hb} + \mathbf{R}_{hb} = 0 \quad (1)$$

$$\frac{d\mathbf{y}_{hb}}{d\tau} + \omega \mathbf{D}\mathbf{y}_{hb} + (\mathbf{A}_s \mathbf{y}_{hb} + \mathbf{B}_s \mathbf{f}) = 0 \quad (2)$$

where  $\mathbf{W}$ ,  $\mathbf{R}$  represent the fluid unknowns and residual;  $\mathbf{y}$  represents the structure displacement, and  $\mathbf{A}$ ,  $\mathbf{B}$  are the state space matrices of the structural problem.  $\mathbf{D}$  is the same HB operator described by:

$$\mathbf{D}_{i,j} = \frac{2}{2n+1} \sum_{k=1}^n k \sin\left(\frac{2\pi k(j-i)}{(2n+1)}\right) \quad (3)$$

Equation (1) together with eq.(2) represent the nonlinear coupled aeroelastic system; when solving the aeroelastic system of equations, at each iteration, the generalized aerodynamic forces are computed using eq.(1), which will feed into eq.(2). The solution from eq.(2) will provide new generalized displacement and grid velocities to eq.(1). To find the LCO condition, eq.(2) is solved for a given combination of  $[\omega, \mathbf{f}]$ , then transfer the displacement



**Figure 1:** Comparison between A-HB and time domain LCO amplitudes and frequencies

back to the fluid system. The frequency is updated by minimizing the residual of eq.2, using the following expression:

$$\frac{\partial \mathbf{L}_n}{\partial \omega} = \left( \omega \mathbf{D} \mathbf{y} - \frac{\partial \mathbf{f}}{\partial \omega} \right)^T [\omega \mathbf{D} \mathbf{y} - (\mathbf{A}_s \mathbf{y} + \mathbf{B}_s \mathbf{f})] \quad (4)$$

If the frequency  $\omega$  is not at the LCO condition, the residual  $\mathbf{R}$  for the displacement is not able to converge.

Initial results show the accuracy of the method for a pitch and plunge aerofoil at transonic conditions, fig.1. The final paper will show results for transonic flows and nonlinear structures and assess their impact on the stability of the aeroelastic system.