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EXPLICIT METHOD SOLVER BASED ON ALTERNATING DIRECTION ISOGEOMETRIC L2 PROJECTIONS

Maciej R. Paszyński[1], Maciej Woźniak[1], Lisandro D. Dalcin[2] and Victor M. Calo[2]

[1] AGH University of Science and Technology, Krakow, Poland,
maciej.paszynski@agh.edu.pl, macwozni@agh.edu.pl,
[2] King Abdullah University of Science and Technology, Thuwal, Saudi Arabia,
dalcinl@gmail.com, victor.calo@kaust.edu.sa

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In this paper we present our new alternating direction preconditioner for isogeometric L2 projections for 2D and 3D problems. The isogeometric finite element method [1] allows us to solve the non-stationary problems with global continuity C^1 that cannot be delivered by classical globally C^0 finite element method.
We propose highly parallelizable alternating direction solver for L2 projections problem. For the parallel distributed memory linux cluster computations, we show that our isogeometric L2 projection solver delivers the following computational cost $\frac{p_x^2 p_y^2 p_z^2 N_x N_y N_z}{U_x U_y U_z} t_{comp} + \frac{(p_x^2+p_y^2+p_z^2) N_x N_y N_z}{U_x U_y + U_y U_z + U_x U_z} t_{comp} + \frac{p_x p_y p_z N_x N_y N_z}{U_x U_y U_z} t_{comp} + \frac{N_x N_y N_z}{U_x U_y + U_y U_z + U_x U_z} t_{comm} +$ for arbitrary polynomial orders p_x, p_y, p_z dimensions N_x, N_y, N_z and processors numbers U_x, U_y, U_z. Here t_{comp}, t_{comm} refer to computational cost of processing a single FLOAT, and communication cost of sending a single byte, respectively.

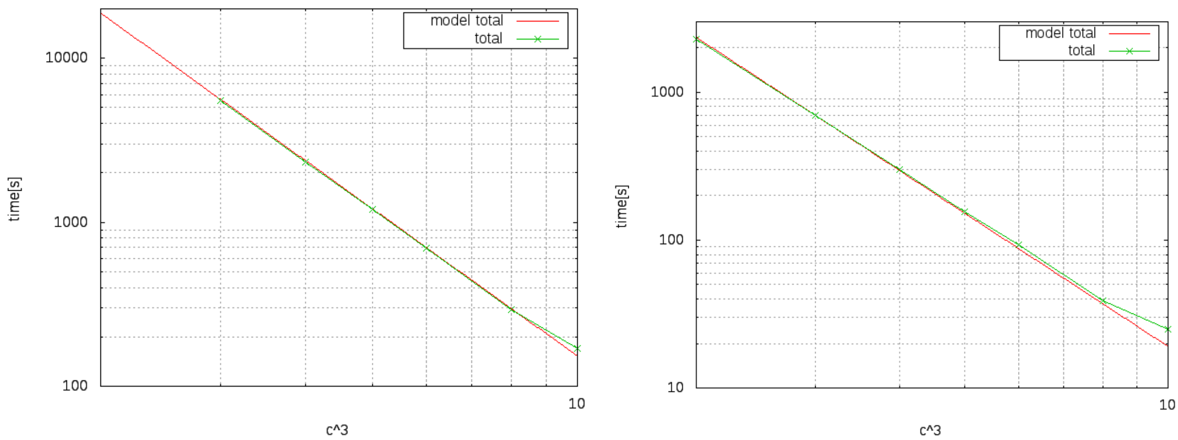

Figure 1. Left panel: The comparison of theoretical and experimental results for alternating direction solver executed over 1024^3 mesh with cubic B-splines, with up to 1000 cores. Using the alternating direction solver we can solve this 10^9 problem within 200 seconds on 1000 cores. **Right panel:** The comparison of theoretical and experimental results for alternating direction solver executed over 512^3 mesh with cubic B-splines, with up to 1000 cores. Using the alternating direction we can solve this 10^8 problem within 10 seconds on 1000 cores.

In particular, if we assume $N_x = N_y = N_z = N$ and $p_x = p_y = p_z = p$ and $U_x = U_y = U_z = U$ we have the following cost

$$\frac{p^6N^3}{U^3} + \frac{p^2N^3}{U^2} + \frac{p^3N^3}{U^3} t_{comp} + \frac{N^3}{U^3} t_{comm} \tag{1}$$

The comparison of the theoretical computational cost with numerical experiments is presented in Figure 1. We consider the application of the parallel L2 isogeometric projection algorithm for explicit methods. The projection problem can be summarized as follows. We start from time dependent problem

$$\begin{aligned} &\frac{\partial u}{\partial t} - L(u) = f(x,t) \quad \text{in} \quad \Omega\times(0,T) \\ &u(x,0) = u_0(x) \quad \text{in} \quad \Omega \end{aligned} \tag{2}$$

where $u = u(x,t)$ real function from $\Omega\times(0,T)$ to R, $L(u)$ linear differential operator with respect to spatial variable x, f real function from $\Omega\times R^+$ to R, u_0 real function from Ω to R, u=0 for $\forall x \in \partial\Omega\times(0,T)$. We start with translating the above strong form into a weak form: For $t\in(0,T)$ find $u\in V$ such that

$$(v,u)_\Omega + b(v,u) = (v,f)_\Omega \quad \forall v \in V \tag{3}$$

where $(f_1, f_2)_\Omega = \int_\Omega f_1\, f_2\, dx$ and $b(v,u)$ is equivalent to $(v, L(u))_\Omega$ plus boundary conditions

The above problem can be written in matrix form as $\mathbf{M\dot{C}} + \mathbf{KC} = \mathbf{F}$ We utilize the Forward Euler Formula $\mathbf{\dot{C}} = \frac{(\mathbf{C}_{n+1} - \mathbf{C}_n)}{\Delta t_n}$, and substitute into matrix form to get

$$\mathbf{M}\frac{(\mathbf{C}_{n+1} - \mathbf{C}_n)}{\Delta t_n} = \mathbf{F}_n - \mathbf{KC}_n \tag{4}$$

The system (3) is equivalent to the L2 projection problem. Our alternating direction L2 projection solver has been implemented within PetIGA [2], the part of PETSc library [3], devoted for isogeometric finite element method computations.

The alternating direction solver enables us to solve a single time step with very high accuracy, with higher order continuity, very efficiently in parallel.

The alternating direction solver can be utilized for solving higher-order non-stationary problems, including the challenging Cahn-Hilliard equations [4].

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