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AUSM like expression of HLLC Scheme and its Extension to All Speed Scheme

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This article deals with a new numerical inviscid flux scheme for MUSCL (Monotone Upwind Scheme for Conservation Laws) in CFD (Computational Fluid Dynamics) for compressible Euler and Navier-Stokes equations which can compute low Mach number flows and sound propagations at the same time with high accuracy, based on HLLC (Harten Lax van-Leer with Contact) scheme.

MUSCL type schemes have been applied for many CFD computations and are the basis of the modern CFD algorithms. The numerical inviscid flux function, sometimes called “Riemann flux”, that calculate the inviscid flux at computational cell boundaries are one of the crucial points in this framework. It has been known that the inviscid numerical flux can be formulated in a simple and robust manner by AUSM (Advection Upstream Splitting Method) family schemes.[1)] The authors also proposed SLAU (Simple Low-dissipative AUSM) type schemes[2,3,4)], those are all speed AUSM family schemes, and they are already applied successfully for various low Mach number flows. SLAU type schemes can also compute acoustic wave at the same time. Such features are favourable to compute low Mach number flows directly coupled to the sound. For aerospace applications, combustion instability in a liquid rocket engine and flows in sound resonators (to suppress the instability) are typical examples. For such applications, we have to treat two phases (liquid and gas), general equations-of-state (EOSs) and so on. We intend to propose a new methodology that can also handle general EOSs of two-phase flows.

Harten, Lax and van Leer invented a way to determine the inviscid flux function by solving the approximate Riemann initial value problem. There are three characteristic speeds in the inviscid fluid, i.e., left and right running acoustic waves and one convective speed. Assuming two constant states divided by three discontinuous waves, they showed that the numerical flux is uniquely defined to reproduce integrated values at both side cells in which the above-mentioned wave structure exists.[5,6)] If three wave speeds are given, the interface flux is given automatically without the need for computing the Jacobian of the flux, and therefore, HLLC scheme can be easily applied for the general EOSs. Low dissipation for low Mach number flows have been also achieved by using numerical sound speed derived from the time derivative preconditioning method[7)], however, such a scheme cannot compute sound propagation.

After some manipulation, HLLC scheme is exactly reformulated in an AUSM-like form, and this notation helps us to understand the “embedded” numerical dissipation of HLLC scheme. The numerical flux by HLLC scheme is expressed exactly as;

$$\widetilde{E}_{HLLC} = \frac{\dot{m}+|\dot{m}|}{2}\mathbf{\Phi}'_L + \frac{\dot{m}-|\dot{m}|}{2}\mathbf{\Phi}'_R + \widetilde{p}\mathrm{N} \tag{1}$$

$$\mathbf{\Phi}'_{L/R} = \begin{pmatrix}\rho & u & v & p & h\end{pmatrix}^T{}_{L/R} + \begin{pmatrix} 0 & 0 & 0 & 0 & \dfrac{\tilde{s}_{L/R}\left(p_* - p_{L/R}\right)}{\rho_{L/R}\left(s_{L/R} - V_{L/R}\right)} \end{pmatrix}^T \tag{2}$$

If the latter part of RHS of Eq.(2) is omitted, the form is identical with AUSM family schemes. It have been known that the crucial points for all speed AUSM family schemes are controlling embedded numerical dissipation in the mass flux $\dot{m}$ and the interface pressure $\tilde{p}$. We will follow the same way for improving HLLC.

It is shown by numerical experiments that the new scheme can compute flows of a wide range of Mach numbers accurately while keeping the ability to compute sound propagations. (Fig.1,2)

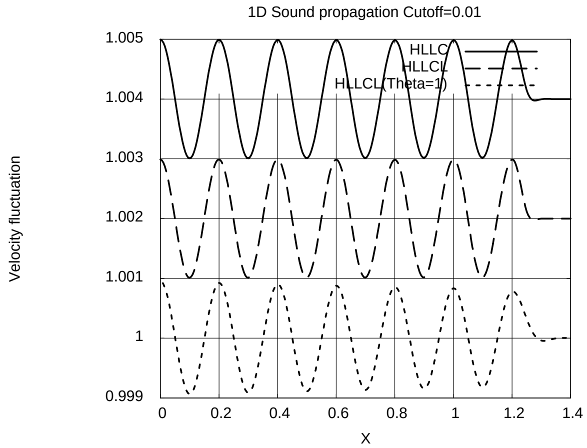

Fig.1 Spatial velocity fluctuation of 1-D sound propagation computed by original HLLC, HLLCL proposed in this pater and HLLCL(θ=1) that correspods to the preconditioned flux scheme. Dissipation of sound propagation is significant in the solution of HLLCL(θ=1).

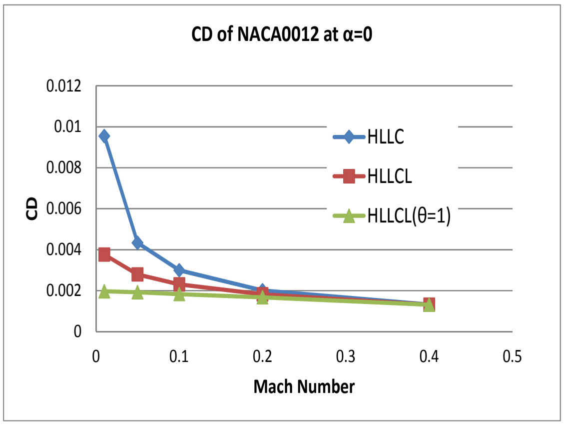

Fig.2 Variation of drag coefficient of inviscid subsonic flows around NACA0012 airfoil due to Mach number. Drag coefficient tends to diverge as Mach number goes down by original HLLC. HLLCL and HLLCL(θ=1) exhibit better results.

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