

# A REGULARIZED NEWTON METHOD FOR THE SOLUTION OF AN INVERSE OBSTACLE SCATTERING PROBLEM IN A FLUID-SOLID INTERACTION

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The determination of the shape of an obstacle from its effects on known acoustic waves is an important problem in many technologies such as sonar, geophysical exploration and medical imaging. This inverse obstacle problem (IOP) is difficult to solve, especially from a numerical viewpoint, because of ill-posed and nonlinear nature. Its investigation requires the understanding of the theory for the associated direct scattering problem, and the mastery of the corresponding numerical solution methods. The main goal of this work is the development of an efficient procedure for retrieving the shape of an elastic obstacle from the knowledge of some scattered far-field patterns, and assuming certain characteristics of the surface of the obstacle. We propose a solution methodology based on a regularized Newton-type method, that can be viewed as an extension of the algorithm designed for solving the case of impenetrable scatterers [1]. The solution of this IOP by the proposed iterative method incurs, at each iteration, the solution of a linear system whose entries are the Fréchet derivatives of the elasto-acoustic field with respect to the shape parameters. Furthermore, ensuring the stability, fast convergence, and computational efficiency calls for computing these derivatives with a greater robustness and accuracy than possible with finite differences. To this effect, following the approach used in [2] for the case of exterior Helmholtz problems, we prove that the required Fréchet derivatives can be characterized as a solution of the same direct elasto-acoustic scattering problem which differs only in the transmission conditions on the surface of the scatterer [3]. Consequently, at each regularized Newton iteration, we can thus evaluate both the scattered field and the directional derivatives by solving a single system of equations with different right-hand sides. Furthermore, the computational efficiency of the IOP solver depends mainly on the computational efficiency of the solution of the forward problems that arise at each Newton iteration. For this reason, we propose to solve the direct scattering-type problems using

a finite-element method based on discontinuous Galerkin approximations equipped with curved element boundaries. Numerical results will be presented to illustrate the potential of the proposed solution methodology for solving efficiently inverse two-dimensional elasto-acoustic problems.

## REFERENCES

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