

## PHASE FIELD MODELING OF BRITTLE AND DUCTILE FRACTURE IN MULTI-PHYSICS ENVIRONMENTS

C. Miehe, H. Ulmer, L. Schänzel

Institute of Applied Mechanics (CE), University of Stuttgart  
Pfaffenwaldring 7, DE-70569 Stuttgart, cm@mechbau.uni-stuttgart.de

**Key words:** *fracture, coupled problems, phase field models, variational principles.*

The lecture provides an overview of recent formulations and computational exploitations of variational principles for dissipative multi-field problems of solids at fracture, which fully govern the coupled initial-boundary-value-problems. This concerns advanced multi-physics scenarios such as chemo-, thermo-, electro-, magneto-mechanics coupled with brittle or ductile crack propagation. In such complex environments, the computational modeling of failure mechanisms based on sharp crack discontinuities suffers in problems with complex crack topologies including branching. This can be overcome by recently developed continuum phase-field approaches to fracture, whose structures fit in a natural format to the multi-physics scenarios. The overall aim of the lecture is to provide a broader understanding of the variational nature of such coupled problems, which includes new aspects of the phase field modeling of ductile and brittle fracture in multi-physics environments. To this end, selected mixed variational principles for representative multi-physics problems of solids are investigated under quasi-static conditions. It is shown that for certain assumptions all coupled equations for the quasi-static evolution and update problems follow in a natural way as the Euler equations of properly defined variational statements. The variational principles derived provide a canonical compact approach to the dissipative multi-physics problems considered. Typical applications of the framework presented are the modeling of complex phenomena such as brittle-to-ductile and ductile-to-brittle fracture mode transitions in thermo-elastic-plastic solids, chemo-mechanical fracture of Li ion batteries or hydraulic fracturing in porous continua. The performances of the proposed formulations are demonstrated by means of representative numerical simulations.

### References

- [1] M. Hofacker and C. Miehe. Continuum phase field modeling of dynamic fracture: variational principles and staggered fe implementation. *International Journal of Fracture*, 178:113–129, 2012.

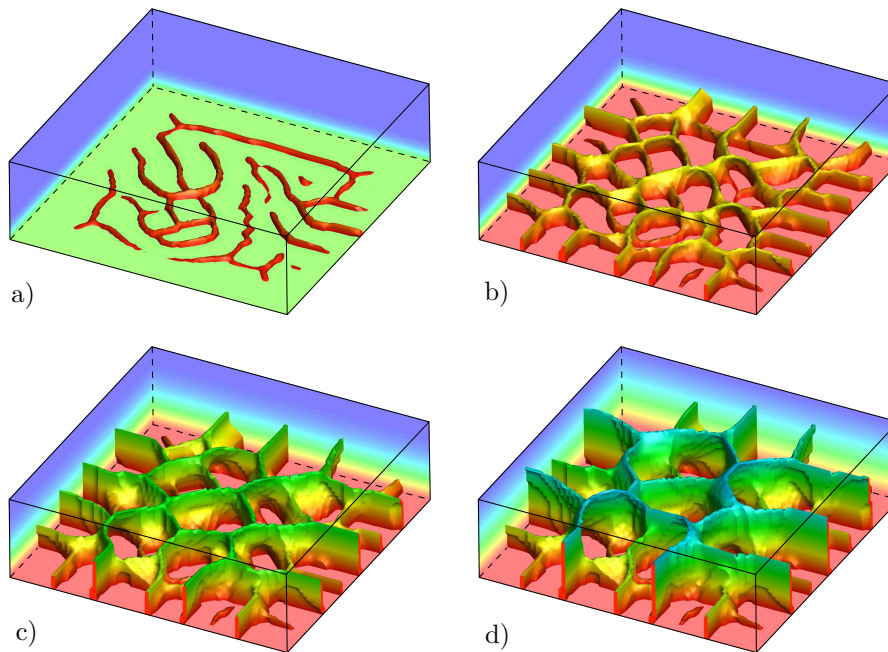


Figure 1: Model problem. Drying soil subjected to linearly decreasing fluid potential at bottom. On the back and bottom of the cuboid the distribution of the fluid content is shown, on the cracks the distance to the bottom is displayed at different times during the drying process.

- [2] M. Hofacker and C. Miehe. A phase field model of dynamic fracture: Robust field updates for the analysis of complex crack patterns. *International Journal for Numerical Methods in Engineering*, 93:276–301, 2013.
- [3] C. Miehe. A multi-field incremental variational framework for gradient-extended standard dissipative solids. *Journal of the Mechanics and Physics of Solids*, 59:898–923, 2011.
- [4] C. Miehe. Variational gradient plasticity at finite strains. part i: Mixed potentials for the evolution and update problems of gradient-extended dissipative solids. *Computer Methods in Applied Mechanics and Engineering*, 268:677–703, 2014.
- [5] C. Miehe, M. Hofacker, and F. Welschinger. A phase field model for rate-independent crack propagation: Robust algorithmic implementation based on operator splits. *Computer Methods in Applied Mechanics and Engineering*, 199:2765–2778, 2010.
- [6] C. Miehe, F. Welschinger, and M. Hofacker. Thermodynamically consistent phase-field models of fracture: Variational principles and multi-field fe implementations. *International Journal of Numerical Methods in Engineering*, 83:1273–1311, 2010.
- [7] C. Miehe, F. Welschinger, and M. Hofacker. A phasefield model of electromechanical fracture. *Journal of the Mechanics and Physics of Solids*, 58:1716–1740, 2010.
- [8] C. Miehe, F. Aldakheel, and S. Mauthe. Mixed variational principles and robust finite element implementations of gradient plasticity at small strains. *International Journal for Numerical Methods in Engineering*, 94(11):1037–1074, 2013.