

RESIDUAL-BASED VARIATIONAL MULTISCALE TURBULENCE MODELS FOR UNSTRUCTURED TETRAHEDRAL MESHES

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Developing stabilized methods for computational fluid dynamics on tetrahedral meshes has been a formidable task, and has therefore attracted considerable attention from the research community. We present a three-level residual-based turbulence model for the incompressible Navier-Stokes equations that is developed for tetrahedral elements. Employing the Variational Multiscale (VMS) framework, the velocity and pressure fields are decomposed into two overlapping hierarchical scales, thereby leading to a system of coupled mixed field problems. The mixed problem at the fine scales is stabilized via a subsequent VMS application that results in a further hierarchical decomposition of the fine-scale velocity field into fine-scales level-I and level-II. The level-II scales are modelled using higher-order bubble functions that are then variationally embedded in the level-I formulation to stabilize it. The level-I problem is modelled via a second set of bubble functions that are linearly independent of the bubbles employed at level-II. A significant feature of the method is that it results in a concurrent and consistent top-down and bottom up two-way nesting of the scales. Another attribute of the formulation is that the fine scales at every level are driven by the residual of Euler-Lagrange equations of the coarser scales at the preceding levels, and therefore the derived formulation is consistent with respect to the governing Navier-Stokes equations. The proposed telescopic depth-in-scales approach results in residual-based models that are accurate for low order tetrahedral and hexahedral elements, a feature that is facilitated by the higher-order bubble functions over element interiors that result in an enhanced representation of fine-viscosity effects. The proposed method results in easy-to-implement equal-order pressure-velocity elements, and possesses the desirable p-refinement feature. Several representative numerical results are presented.

REFERENCES

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