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ENHANCED FORMULA FOR A CRITICAL VELOCITY OF A UNIFORMLY MOVING LOAD INCLUDING SHEAR CONTRIBUTION

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The response of rails to moving loads is of interest in the area of high-speed railway transport. For determination of critical velocity of the train a theoretical concept that is based on the assumption that the track structure acts as a continuously supported beam (the rail) resting on a uniform layer of springs, is traditionally used. Then the critical velocity of the load is given by the classical formula [1]:

$$v_{cr}^{E-B} = \sqrt[4]{\frac{4kEI}{m^2}} \qquad (1)$$

where *E-B* designates that Euler-Bernoulli theory is used, m stands for the beam (rail) mass per unit length, EI for the beam bending stiffness and k for the track modulus, i.e. for the Winkler constant of the foundation, or stiffness of the underlying remainder of the track structure. The same result is obtained when the concept of the dynamic stiffness matrix is implemented. Then the quasi-stationary deflection shape of an infinite beam can be determined from two semi-infinite beams and critical velocities can be obtained from the nullity condition of the determinant of the dynamic stiffness matrix of the structure. Similar concept is defined for finite beams [2, 3].

It is seen that the inertia of the load and of the foundation are neglected in formula (1). It can be proven that in the steady-state regime load exerts no inertial effects [1], which is probably the reason why also the mass inertia of the foundation was overlooked. Unfortunately, practical experience showed that the realistic critical velocity is much lower. The reason for this lies mainly in the simplifications adopted for the foundation model. In order to improve the formula (1), two important notions must be introduced: an effective finite depth H of the foundation that is dynamically activated and an inertial effect of this activated foundation layer.

Vlasov and Modified Vlasov foundation model account for a finite foundation depth [4]. Distribution of the vertical displacement is estimated by a concave shape function f that rapidly decreases with the increasing depth. In a finite element analysis the foundation mass is usually added directly to the mass matrix of the full structure, assuming inconsistently a linear shape function.

In the new approach, which preliminary results were presented in [5], the dynamic equilibrium of the soil in the vertical direction is implemented to obtain two frequency dependent parameters characterising the inertia of the dynamically activated soil layer, namely, Winkler (normal, vertical) and Pasternak (shear or rotational), similarly as in [6]. Unfortunately, there is an incompatibility caused by several simplifying assumptions and thus only the frequency dependent vertical stiffness can be safely used. This is in accordance with solution presented in [7] for infinite beams, nevertheless, unlike in [7], a closed form solution is derived in this contribution. Then the critical velocity, v_{enh}^{E-B}, can be estimated as:

$$v_{enh}^{E-B} = \sqrt[4]{\frac{4k_{st}EI}{\tilde{\mu}^2}}\,,\ \tilde{\mu} = \mu + \varsigma\rho bH\,,\ \varsigma = \frac{1}{H}\lim_{\lambda H\to\pi/2}\left(\int_{z=0}^{H} f^2\left(z\right)dz\right) \approx 0.5 \tag{2}$$

where k_{st} is the static Winkler constant, ρ is the soil density and b is the soil strip width. An alternative approach is proposed to include the shear contribution.

The analysis of finite beams is based on modal expansion method, while the solution for infinite beams is attained via Fourier transform. Results obtained are analysed and compared. Conclusions about the enhanced formula for the critical velocity are drawn.

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