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OPTIMAL TRACKING CONTROL OF ROTATING MULTI-TETHERED FORMATIONS IN HALO ORBITS

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Spacecrafts moving on their paths must use some form of trajectory control to remain close to their nominal orbits because libration point orbits are inherently unstable in general. A series of papers has presented results from studies that use Floquet and invariant manifold theories to develop a “loose” station-keeping strategy for halo orbits [1]. Most recently, Kullarni et al. [2] extended the traditional H_∞ framework to periodic discrete linear time-varying (LTV) systems to solve the problem. In the current study, an approach called “nonlinear optimal tracking control” based on optimal control theory is applied to the orbit control problem of multi-tethered satellite formations in larger Halo orbits.

An “Approximating Sequence of Riccati Equations” (ASRE) method is introduced by Tayfun and Banks [3] to find time-varying feedback controllers for nonlinear systems. This reduces the nonlinear problem to a sequence of linear-quadratic and time-varying approximating problems. Suppose that the system can be described in the factored control-affine form by the equations

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}\mathbf{u}, \mathbf{x}(t_0) = \mathbf{x}_0, \ \ \mathbf{y} = \mathbf{C}\mathbf{x} \tag{1}$$

together with the finite-time nonlinear cost function

$$J = \frac{1}{2}\int_0^{tf}\left[\mathbf{e}^T\mathbf{Q}\mathbf{e} + \mathbf{u}^T\mathbf{R}\mathbf{u}\right]dt + \frac{1}{2}\mathbf{e}^T\left(t_f\right)\mathbf{S_f}\mathbf{e}\left(t_f\right), \ \ \mathbf{e} = \mathbf{z} - \mathbf{y} = \mathbf{z} - \mathbf{C}\mathbf{x} \tag{2}$$

where superscript T denotes the transpose and $\mathbf{x}$ is a state vector. The weighting matrix $\mathbf{R} \in \mathbb{R}^{m\times m}$ is symmetric positive-definite, and $\mathbf{S_f}, \mathbf{Q} \in \mathbb{R}^{l\times l}$ are both positive-semidefinite. $\mathbf{u}$ is unconstrained, $\mathbf{A} \in \mathbb{R}^{n\times n}, \mathbf{B} \in \mathbb{R}^{n\times m}$, $\mathbf{C} \in \mathbb{R}^{l\times n}$, and the objective is to control the system Eq. (1) so that the output vector $\mathbf{y}$ is “near” the desired output vector $\mathbf{z}$. An ASRE method reduces Eq. (1) and (2) to a sequence of linear-quadratic and time-varying approximations. For the number of approximation sequences $i_s \geq 0$, the optimal tracking control law for the nonlinear problem Eq. (1) and (2) can be given in the form

$$\mathbf{u}^{[i_s]} = -\mathbf{R}^{-1}\mathbf{B}^T\left(\mathbf{x}^{[i_s-1]}\right)\left\{\mathbf{P}^{[i_s]}\mathbf{x}^{[i_s]} - \mathbf{b}^{[i_s]}\right\} \tag{3}$$

where the symmetric and positive-definite matrix $\mathbf{P}^{[i_s]}(t)$ is the solution of the ASRE, and the vector $\mathbf{b}^{[i_s]}(t)$ is the solution of the linear vector differential equation. The optimal trajectory becomes the limit of the solution of the linear differential equation

$$\dot{\mathbf{x}}^{[i_s]} = \left[\mathbf{A}\left(\mathbf{x}^{[i_s-1]}\right) - \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}^{[i_s]}\right]\mathbf{x}^{[i_s]} + \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{b}^{[i_s]}, \quad \mathbf{x}^{[i_s]}(t_0) = \mathbf{x}_0 \tag{4}$$

In solving the set of coupled Eqs. (1)-(4), optimization must be carried out on the system trajectory at every numerical integration time-step for each sequence, resulting in time-varying feedback controls. The precise integration method [4], which is based on the theory of analogy between structural mechanics and optimal control, can be employed to solve the linear time-varying differential equation of each sequence precisely and efficiently instead of by using the usual finite difference method. The controlled trajectories, position tracking errors and control inputs are presented by numerical simulations. We show that the proposed nonlinear optimal tracking control law is adequate for the station-keeping of multi-tethered satellite formations while taking into account the initial injection errors and perturbations of subsatellites. By using this approach, the allowable deviation of the actual trajectory relative to the nominal path can be varied over a wide range depending on mission requirements. The thrust required by the control law is also reasonable and can be implemented using a low-thrust propulsion device such as an ion engine. Simultaneously, low costs are desirable.

The robust performance of the controller is tested by varying some parameters related to the controller, including the orbit characteristics and the tethered subsatellites. Several parameters of concern are highlighted and potential ideas for improvement are suggested. This controller depends heavily on the number of ASRE and the tracking interval. A higher ASRE number and a shorter tracking interval provide a considerable reduction in the tracking errors and control inputs, which also corresponds to more CPU time for running the controller. It seems that the control performance gains no obvious improvement by further increasing the number of ASRE or decreasing the tracking interval beyond a certain limit. Consequently, it is evident that we must find a balance between the control effect and the CPU time by choosing the suitable number of ASRE approximations and a suitable tracking interval. Additionally, the orbital amplitude, the orbital direction of the parent satellite and the mass ratios of the subsatellites are varied in order to demonstrate the robustness of the nonlinear controller. The control performance is affected to a great extent as the mass ratios and the orbital amplitude are varied, but performance is not significantly affected by the orbital directions. Using the nonlinear control techniques here. the Halo orbit not with an orbital amplitude of 200,000km but with an orbital amplitude of 400,000km appears to produce the lowest total cost and the least deviations, which is entirely different results based on local linearization techniques.

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