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MYOCARDIAL TISSUE MECHANICS WITH FIBRES MODELLED AS ONE-DIMENSIONAL COSSERAT CONTINUA

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1 Introduction

Classically, the elastic behaviour of cardiac tissue mechanics is modelled using anisotropic strain energy functions capturing the averaged behaviour of its fibrous microstructure. The strain energy function can be derived via representation theorems for anisotropic functions where a suitable nonlinear strain tensor, e.g. the *Green* strain tensor, describes locally the current state of strain. These kinds of approaches, however, are usually of phenomenological nature and do not elucidate on the complex heterogeneous material composition of cardiac tissue characterized by different fibre hierarchies interwoven by collagen, elastin and coronary capillaries. Thus, pathological changes of microstructural constituents, e.g. with regards to the extra-cellular matrix, and their implications on the macroscopically observable material behaviour cannot be directly investigated.

This paper follows a hypothesis by Hussan et al. [1], stipulating that the semisoft behaviour of myocardial tissue stems from the ability of cardiac myocytes to deform while being embedded and constrained by the cross-linked collagen matrix. Here, the fibrous characteristics of the myocardium are modelled by one-dimensional *Cosserat* continua. This additionally allows for the inclusion of fibre motion relative to matrix representing the non-local material response due to twisting and bending of fibres.

2 Method

We decompose the strain energy in contributions related to fibrous structural components, e.g. bundles of myocytes embedded in perimysial collagen, modelled by one-dimensional

Cosserat continua and complementary connective tissue. In the *Cosserat* continuum every material point P is assigned besides displacement additional rotational degrees of freedom described by a rotation tensor $\mathbf{R} \in SO(3)$ which is independent of the deformation gradient $\mathbf{F}$. This allows for the formulation of two strain measures, a stretch-like strain tensor $\mathbf{U} = \mathbf{R}^T\mathbf{F}$ and a change of curvature strain tensor $\mathbf{K} = -\frac{1}{2}\boldsymbol{\epsilon} : (\mathbf{R}^T\mathbf{R}_{,i}) \otimes \mathbf{V}_i$ where the index $i = f, t, n$ and $\mathbf{V}_f, \mathbf{V}_t, \mathbf{V}_n$ span an orthonormal basis describing fibre, sheet-tangent and sheet-normal directions, respectively.
A *Fung*-type orthotropic strain energy function $\psi(\mathbf{U}, \mathbf{K})$ incorporating fibrous and complementary matrix material components is defined as follows:

$$\psi = \frac{1}{2} A \left(\exp^{BQ} - 1 \right) + A_{comp} \, \left(J \, \ln J - J + 1 \right) \tag{1}$$

with J denoting the Jacobian and the exponent $Q = Q_{fibre} + Q_{matrix}$ given by

$$\begin{aligned} Q_{fibre} &= a_1 \, U_{ff}^2 + a_2 \, \left(U_{ff}^2 + U_{tf}^2 + U_{nf}^2 \right) + \sum_i^N \left\{ a_3^{(i)} \, K_{ff}^2 + a_4^{(i)} \, \left(K_{ff}^2 + K_{tf}^2 + K_{nf}^2 \right) \right\} \\ Q_{matrix} &= b_1 \, \left(U_{tt}^2 + U_{nn}^2 \right) + b_2 \, \left(U_{ft}^2 + U_{tt}^2 + U_{nt}^2 + U_{fn}^2 + U_{tn}^2 + U_{nn}^2 \right) \end{aligned}$$

considering the heterogeneous responses of N different hierarchical fibre entities and the associated material parameters A, B, A_{comp}, a_1, a_2, $a_3^{(i)}$, $a_4^{(i)}$, b_1 and b_2.
Finally, a corresponding variational principle is formulated as

$$\int_{\mathcal{B}} \{ \boldsymbol{n} : \delta\mathbf{U} + \boldsymbol{m} : \delta\mathbf{K} \} \, \mathrm{d}V - W_{ext} = 0 \tag{2}$$

with the force stress tensor $\boldsymbol{n} = \dfrac{\partial \psi}{\partial \mathbf{U}}$ and the couple stress tensor $\boldsymbol{m} = \dfrac{\partial \psi}{\partial \mathbf{K}}$. The latter incorporates the heterogeneous responses of all considered fibrous constituents.

3 Conclusion

In summary, the proposed approach is motivated by the microstructural kinematics of myocytes and bundles of myocytes interacting with the collagen enmeshment. It allows for the description of the mechanics of both constituents and pathological changes thereof in a more detailed fashion.

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