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3D CONTINUUM MODELS OF TENSEGRITY MODULES WITH THE EFFECT OF SELF-STRESS

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The objective of the present paper is to describe the continuum model of the properties of tensegrity modules with the effect of self-stress included. 3D tensegrity modules with similar dimensions in each direction are considered. The term of tensegrity was introduced by Fuller (see [6] for historical details). There are several definitions of this concept [4]. For the purpose of this paper the tensegrity is defined as a pin-joined system with a particular configuration of cables and struts that form a statically indeterminate structure in a stable equilibrium. Infinitesimal mechanism should exist in a tensegrity with equivalent self-stress state. Major advantages of tensegrity are: large stiffness-to-mass ratio, deployability, reliability and controllability [4,6].

The continuum model of tensegrity modules should make possible to:

- Estimate properties of the module with typical deformation modes (tension, shear),
- Evaluate the influence of self-stress for the defined deformation,
- Evaluate the influence of cabels and struts for the properties of the module,
- Compare the elastic properties of typical tensegrity modules.

The above estimations can help to build complex tensegrity based beam- plate or shell- like structures ([5] for lattice models). The open problem is how to connect the modules together.

Discrete models.

Discrete models of tensegrity modules are mathematically described with the use of the Finite Element Method [7]. Strain energy is a quadratic form $E_s^{Tensegrity-FEM} = \frac{1}{2}\mathbf{q}^T\mathbf{K}\mathbf{q}$ of nodal displacements $\mathbf{q}$ with the global linear and geometric stiffness matrix $\mathbf{K} = \mathbf{K}_L + \mathbf{K}_G$ as a kernel. The self-stress state of the module (proportional to the tension force S) is represented by the geometric stiffness matrix.

Continuum model.

Symmetric linear 3D elasticity theory is considered. The strain energy can be expressed as $E_s^{Elasticity} = \frac{1}{2}\int_V \boldsymbol{\varepsilon}^T\mathbf{E}\boldsymbol{\varepsilon}dV$, where $\boldsymbol{\varepsilon}$ - is the strain vector, $\mathbf{E}$ – is the elasticity matrix.

It is assumed in the proposed concept that the strain energy of not supported tensegrity is equivalent to the strain energy of the sphere (radius R) – Fig. 1 – with constant strains.

To compare the energies and build the equivalent matrix $\mathbf{E}$ the nodal displacements are expressed by the average mid-values of displacements and their derivatives with the use of

Tayor series expansion. Co-ordinates of nodal points $\{\alpha_{xi} \cdot R, \alpha_{yi} \cdot R, \alpha_{zi} \cdot R,\}$ are expressed by the radius R with the increments $\Delta x = \alpha_{xi} \cdot R$, $\Delta y = \alpha_{yi} \cdot R$, $\Delta z = \alpha_{zi} \cdot R$ in Taylor series .

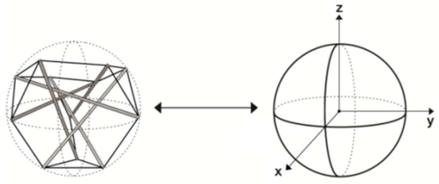

Figure 1. Tensegrity and continuum.

Elasticity tensor can be expressed in Voight's form $\mathbf{E} = [E_{ij}]$ *i,j-1,2,…6.* There are 21 independent co-efficients for anisotropy. One of eight symmetries of the tensor **E** can be expected with 13, 9, 6, 5, 3 or 2 independent co-efficients, respectivelu [1].

Examples

The proposed technique can be used for any tensegrity configuration. Some typical modules (3 and 4-strut Simplex, Expanded and Truncated Octahedron) are presented in Fig. 2.

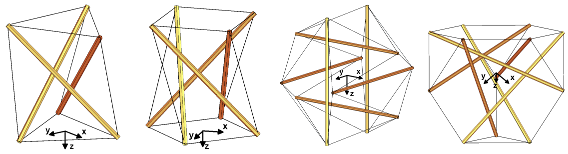
Figure 2. Typical tensegrity modules.

As an example, the coefficients received for 4-strut Simplex module are presented below

$$e_{11} = e_{22} = \frac{EA}{4/3\, R^2}(3.8419 + 1.46386 \cdot k - 0.121763 \cdot \sigma),$$

$$e_{33} = \frac{EA}{4/3\, R^2}(2.31641 + 0.59533 \cdot k - 0.324703 \cdot \sigma),$$

$$e_{12} = e_{44} = \frac{EA}{4/3\, R^2}(1.28063 + 0.487953 \cdot k - 0.0405878 \cdot \sigma),$$

$$e_{13} = e_{55} = e_{66} = \frac{EA}{4/3\, R^2}(0.508845 + 0.76233 \cdot k + 0.162351 \cdot \sigma),$$

$$e_{14} = -e_{24} = \frac{EA}{4/3\, R^2}(0.055889 - 0.487953 \cdot k - 0.193724 \cdot \sigma),$$

$$e_{15} = e_{16} = e_{25} = e_{26} = e_{35} = e_{36} = e_{45} = e_{46} = e_{56} = 0,$$

where $k = \frac{(EA)_{Strut}}{(EA)_{Cable}}$, $(EA)_{Cable} = EA$, $\sigma = \frac{S}{EA}$.

Detailed discussion of the results for various tensegrity modules will be presented during the Conference with some reccomendations for beam-, plate- and shell-like tensegrity structures.

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