11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

HIGH ORDER DISCONTINUOUS GALERKIN METHODS FOR LARGE EDDY SIMULATIONS

A. D. Beck*[1], G. J. Gassner[2] and C.-D. Munz[3]

[1,3] Institute of Aerodynamics and Gas Dynamics, University of Stuttgart, Pfaffenwaldring 21, 70569 Stuttgart, Germany,
[2] Mathematical Institute, University of Cologne, Weyertal 86-90, 50931 Cologne, Germany

Key words: *High Order Schemes, Discontinuous Galerkin Spectral Element Method, De-aliasing, Over-integration, Large Eddy Simulation*

Since its introduction in the 1970s, the family of Discontinuous Galerkin (DG) methods has been extended continuously and has reached a certain level of maturity that makes it an attractive candidate for the simulation of complex multi-scale and multi-physics problems, governed by systems of evolution equations like the Navier-Stokes-Fourier system or the equations of magnetohydrodynamics. In recent years, a strong interest in combining DG with time-averaged turbulence simulations (Reynolds-Averaged Navier-Stokes, RANS) has arisen, and a number of successful implementations and computations have been reported, e.g. in [1, 2]. In contrast, the extension of DG methods to Large Eddy Simulation type problems is somewhat lagging behind, although high order DG methods with their high wave resolution capabilities per degree of freedom, their operator structure that supports excellent parallel scaling and their robustness for advection-dominated problems make an attractive candidate as base LES methods [3].

One issue that arises for non-linear multi-scale problems in under-resolved settings is the numerical treatment of the scale-producing mechanism, i.e. the removal or avoidance of aliasing errors. While the inherent numerical dissipation of low order schemes may be sufficient to dampen the effects of aliasing, high order schemes mandate a form of de-aliasing. Typical examples include the exact evaluation of the inner products (termed over-integration, [6]) or filtering. For the same number of degrees of freedom, properly de-aliased high order formulations have been shown to clearly outperform their low-order counterparts for under-resolved computations of turbulent flows [4].

In this work, we will present the results of under-resolved high order computations ($N \geq 7$) of the Discontinuous Galerkin Spectral Element Method [5] to canonical turbulence test cases like the flow over a circular cylinder at $RE_D = 3900$. We will show that while aliasing-afflicted computations may be stabilized by a naive combination with an explicit subgrid scale model, the solution quality suffers greatly, as the role of this model becomes aliasing control instead of turbulence closure. Fig. 1 depicts a comparison of high order LES approaches with a DNS result: While the de-aliased "no-model" solution is in good

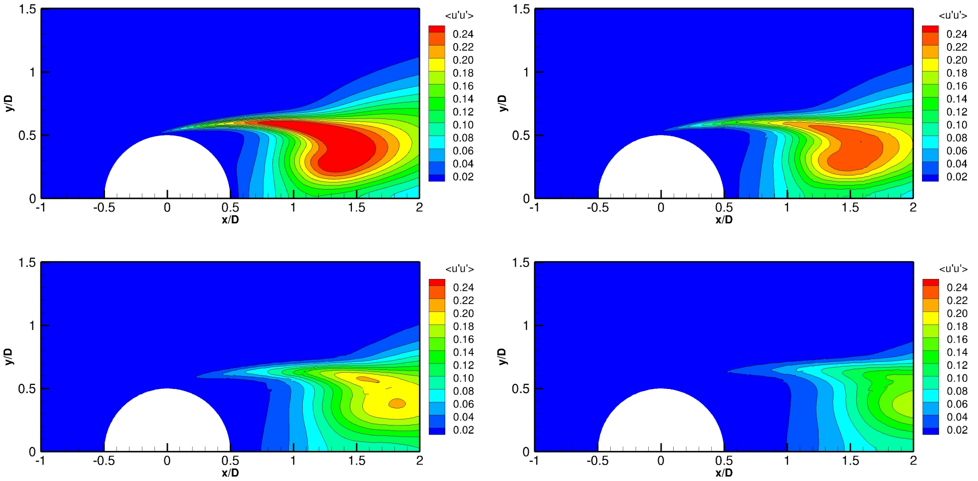

Figure 1: Time- and spanwise-averaged velocity fluctuations $\langle u'u' \rangle$ of $RE_D = 3900$ cylinder flow. *Upper row:* DNS results (*left*), de-aliased $N = 7$ computation without SGS model (*right*), *Lower row:* $N = 7$ computation with Smagorinsky model ($C_S = 0.14$) (*left*), $N = 7$ computation with Smagorinsky model ($C_S = 0.16$) (*right*). Total number of degrees of freedom for DNS and LES: $\approx$ 200mio and $\approx$ 3mio.

agreement with the DNS, the solution quality deteriorates strongly for not properly de-aliased solutions with an added explicit Smagorinsky LES closure. Our findings therefore indicate two important issues: Firstly, properly de-aliased high-order DG schemes may produce accurate results for medium Reynolds number regimes even without additional LES modeling and secondly a control of the aliasing errors is mandatory not only for stability reasons but also to avoid interference with any form of turbulence closure.

REFERENCES

[1] F. Bassi, A. Crivellini, S. Rebay, M. Savini. Discontinuous Galerkin Solution of the Reynolds-averaged Navier Stokes and k$-\omega$ Turbulence Model Equations. *Computers & Fluids*, Vol. **34**, 507 - 540, 2005.

[2] N. Nguyen, P.-O. Persson, J. Peraire. RANS Solutions using High Order Discontinuous Galerkin Methods. *AIAA*, Vol. **914**, 2007.

[3] A. Beck, T. Bolemann, D. Flad, H. Frank, G. Gassner, F. Hindenlang, C.-D. Munz. Stabilized High Order DGSEM for Turbulent and Detached Flow Simulations. *International Journal for Numerical Methods in Fluids*, Submitted, 2013.

[4] G.J. Gassner, A.D. Beck. On the Accuracy of High-Order Discretizations for Underresolved Turbulence Simulations. *Theoretical and Computational Fluid Dynamics*, Vol. **27**, 221 - 237, 2013.

[5] D. A. Kopriva. *Spectral Element Methods*. Implementing Spectral Methods for Partial Differential Equations, 2009.

[6] R.M. Kirby, G.E. Karniadakis. De-aliasing on Non-uniform Grids: Algorithms and Applications. *Journal of Computational Physics*, Vol. **191**, 249 - 264, 2003.