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METHOD OF DATA COMPLETION FOR A CAUCHY PROBLEM OF ELASTOPLASTICITY BASED ON MINIMIZING AN ENERGY ERROR FUNCTIONAL

Thi Bach Tuyet Dang [1], Thouraya Nouri Baranger[2] and Stéphane Andrieux[3]

[1] INSA de Lyon, LamCoS UMR5259, 69621 Villeurbanne cedex, France,
thi-bach-tuyet.dang@insa-lyon.fr
[2]Université de Lyon, CNRS, Université Lyon 1, LamCoS UMR5259, 69621 Villeurbanne cedex, France, Thouraya.baranger@univ-lyon1.fr
[3] LaMSID, UMR EDF-CNRS-CEA 8193, 1, avenue du Général du Gaulle, 92141 Clamart cedex, France, Stephane.Andrieux@edf.fr

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ABSTRACT

In previous works, two of the authors introduced a method for solving the ill-posed Cauchy problem by minimizing the so-called energy error functional, see [1, 2, 3, 6, 5, 4]. This method consists in solving an optimisation problem under constraints. It has proved its numerical performance, especially, being easier in practical implementation, saving the calculating time and enabling applications in 3D heterogeneous domains, see [4, 6]. In addition, the most importance here is that the method is not only restricted to the case of linear problem, but also can be generalized for nonlinear problems in the context of convex functional. In the present study, we would like to present its general validation in both nonlinear constitutive laws: hyperelasticity and elastoplasticity. The principal idea of the method here is to construct two independent well-posed problems which are derived from the original ill-posed one, then to set up a non-negative energy error functional from the two well-posed problem solutions. This functional quantifies the gap between the solutions of those problems. Hence, the missing data are the arguments of the energy error functional. Our work proved that the exact solution is the one that vanishes the error functional in both hyperelasticity and elastoplasticity. Therefore, that triggers a noticeable problem of choosing a suitable method is to achieve the minimizer optimally. In addition, the accumulations of errors by increment calculations for non-linear problems, particularly, plastic problems is inevitable. Therefore, the method for numerical approximations should be well-chosen in order to minimize the errors at each step of increment. Application for an elastoplastic hollow sphere under internal pressure history both for perfect and hardening plasticity will be given together with a full 2D example.

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