11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

BAYESIAN UPDATING OF NUMERICAL MODELS WITH SUBSET SIMULATION

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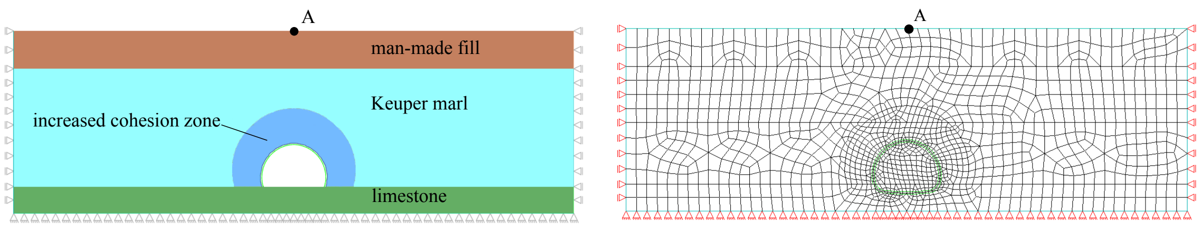
Key words: *Model Calibration, Bayesian Updating, Reliability Analysis, BUS, Subset Simulation, MCMC, Finite Elements.*

Calibration of numerical models corresponds to determining the model parameters that provide the best match between the models and experimental observations. In probability theory, the process of learning model parameters from observations is formalized in the concept of Bayesian updating. Thereby, the prior probability distribution of the model parameters is updated with new data and information to a posterior distribution.

Bayesian updating requires the solution of a potentially high-dimensional integral to obtain the posterior distribution of the model parameters. Commonly, Markov Chain Monte Carlo (MCMC) sampling is applied to sample directly the posterior distribution, thus bypassing the solution of the aforementioned integral. The disadvantage of MCMC is the difficulty in ensuring the stationarity of the Markov chain. An alternative approach was recently proposed based on interpreting the Bayesian updating problem as a structural reliability problem [1, 2]. This approach, termed Bayesian Updating with Structural reliability methods (BUS), applies methods originally developed for structural reliability analysis to represent the posterior distribution.

We apply BUS in conjunction with subset simulation, which is an adaptive Monte Carlo method originally developed in [3] for the estimation of small failure probabilities in high-dimensional problems. Subset simulation expresses the probability of failure as a product of larger conditional probabilities, which are estimated by application of MCMC. It is noted that the initial samples of the Markov chains follow the target conditional distribution by construction, hence convergence of the chains is not an issue. We employ an efficient MCMC algorithm proposed in [4] for application within subset simulation. BUS with subset simulation is shown to be efficient in producing samples of high-dimensional posterior distributions [1].

We demonstrate the performance of subset simulation in the Bayesian updating of a finite element model of a tunnel in soft soil conditional on settlement measurements. We model the tunnel in two dimensions using plain strain finite elements. Figure 1 shows the ground layers and the finite element mesh. The nonlinear behavior of the ground is modeled with a hardening plasticity soil model. The prior distribution of the soil parameters is modeled with non-Gaussian homogeneous random fields. We assume that a measurement of the surface settlement (point A in Fig. 1) is taken at full excavation and utilize the measurement to update the distribution of the parameters of the numerical model. As an example, the posterior sample mean of the oedometric stiffness E_{oed} of the Keuper marl layer is displayed in Figure 2.



(a) Ground layers considered in the model.

(b) Finite element mesh.

Figure 1: Numerical model of a tunnel in soft soil.

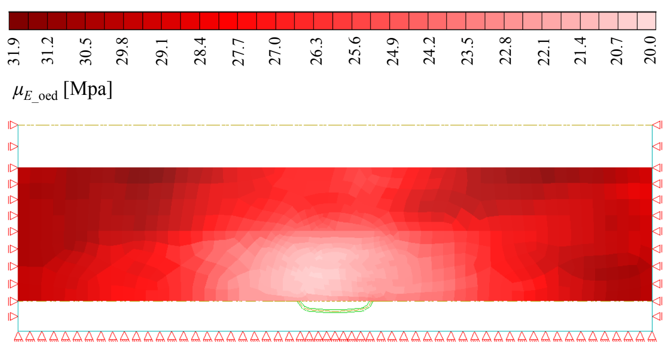


Figure 2: Posterior mean of the oedometric stiffness modulus E_{oed} of the Keuper marl layer.

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