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A 3-D Mach-Uniform Preconditioner for Incompressible and Subsonic Flows

Onur BAS[1] and Ismail H. TUNCER[2]
[1]Phd Student, Aerospace Engineering, METU
e-mail: onur.bas@tai.com.tr

[2]Professor, Aerospace Engineering, METU
e-mail: tuncer@ae.metu.edu.tr

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ABSTRACT

In recent years, beyond various different approaches, preconditioning methods gained increasing popularity for all speed flow solver development studies. The resulting formulation of these methods is very similar to the well-known compressible governing equations which allow the usage of widely used numerical methods and algorithms. On the contrary to pressure based methods, preconditioning methods are also very successful on high speed flows with their conservative form equation set.

In preconditioning methods, the time derivative terms of the Euler equations are pre-multiplied by a matrix for relaxation to enhance the convergence behavior without altering the steady-state solution. One of the pioneering researches on low speed flows is Artificial Compressibility Method and accepted to be the first preconditioning methods. Several improvements and extensions are later developed and families of preconditioners are introduced.

In this study, a recently developed novel preconditioning formulation is extended to three dimensional problems. The major difference of the developed formulation is usage of conservation of energy equation to enforce divergence free flow field on low speeds. Despite the general approach of preconditioning the continuity equation, it is both theoretically and numerically proved that proper preconditioning of the energy equation can also ensure the continuity. The resulting equation set behaves as the non-preconditioned conservative form of compressible Euler formulation on high Mach number flows and converges to well-known Artificial Compressibility Formulation at the incompressibility limit. The resulting preconditioned equation set is shown below.

$$\frac{\partial Q}{\partial t} + \Gamma\left(\frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial G}{\partial z}\right)$$

$$Q = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho e \end{pmatrix} \quad E = \begin{pmatrix} \rho u \\ \rho u^2 + P \\ \rho uv \\ \rho uw \\ u\left(\rho e + P + \frac{1}{(\gamma-1)M_\infty^2}\right) \end{pmatrix} \quad F = \begin{pmatrix} \rho v \\ \rho vu \\ \rho v^2 + P \\ \rho vw \\ v\left(\rho e + P + \frac{1}{(\gamma-1)M_\infty^2}\right) \end{pmatrix}$$

$$G = \begin{pmatrix} \rho w \\ \rho w u \\ \rho w v \\ \rho w^2 + P \\ w\left(\rho e + P + \dfrac{1}{(\gamma-1)M_\infty^2}\right) \end{pmatrix}$$

$$\Gamma = \begin{bmatrix} M_\infty^2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & (1-M_\infty^2)u & (1-M_\infty^2)v & (1-M_\infty^2)w & M_\infty^2 \end{bmatrix}$$

On the present study, the proposed formulation is implemented into an unstructured parallel in-house code and validated on 3-D external flows for several Mach numbers. The convergence character is compared against non-preconditioned compressible Euler equations on ONERA M6 test case. The preliminary results show that Mach uniformity is accomplished whereas convergence character is not as good as non-preconditioned formulation. The convergence rate will be ameliorated with extensive work on boundary conditions, discretization and coding.

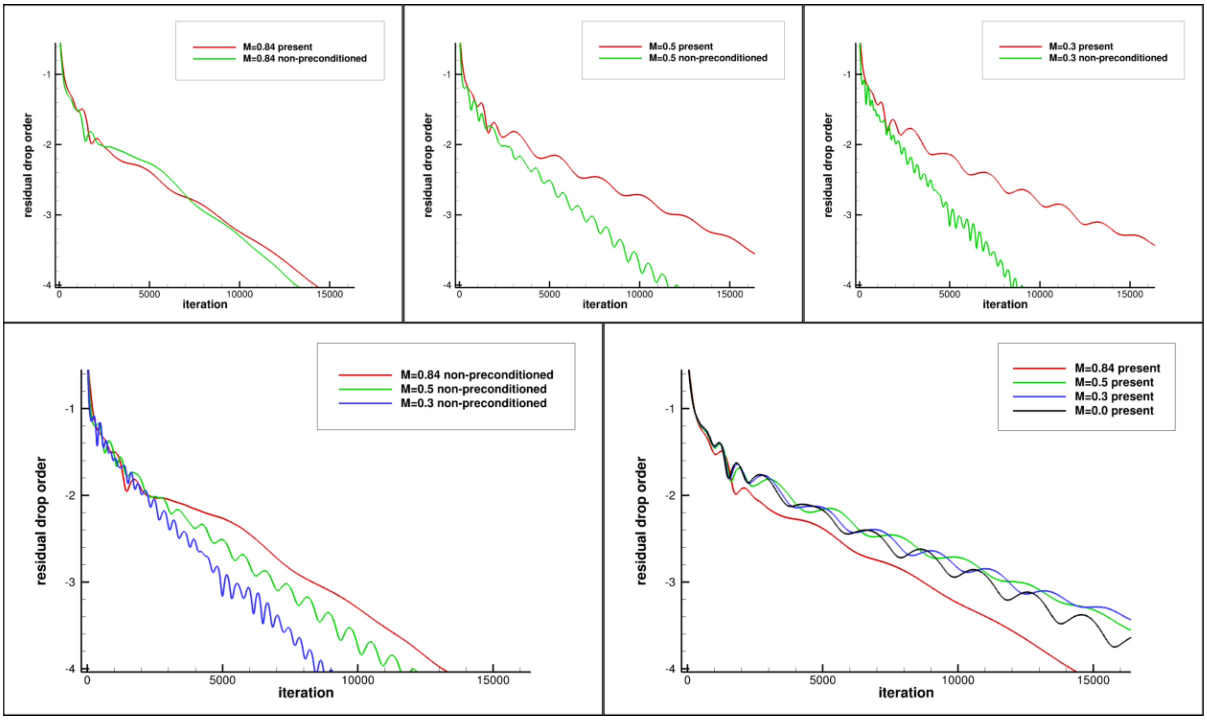

Figure 1. Convergence Graphs of Present and Non-Preconditioned Formulations

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