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ON STRAIN LOCALIZATION UNDER BENDING

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Realistic mathematical description of quasibrittle materials based on pure continuum models (without displacement discontinuities) requires constitutive laws with softening, which often leads to localization of inelastic processes into narrow bands. If the standard local continuum approach is used, the thickness of the numerically resolved localized inelastic bands depends on the size of finite elements or on similar discretization parameters, and the solution exhibits pathological dependence on the computational grid (e.g., finite element mesh). This can be remedied by simple adjustments that make some of the constitutive parameters depend on the element size, or by more sophisticated enhancements that introduce an internal length, e.g. based on integral-type or gradient-type nonlocal formulations [1, 2, 3].

In previous studies, much has been learned about localization properties of nonlocal formulations of plasticity, damage mechanics, smeared crack models, or microplane models; see e.g. [4] for an overview. For uniaxial tension modeled in the one-dimensional setting, analytical or semi-analytical solutions can be developed and the thickness of the localized inelastic band can be directly linked to the internal length and other material parameters and, in some cases, also to the length of the tensile bar [5, 6]. In many computational studies and applications, it has been demonstrated numerically that the band thickness remains finite and independent of the element size even for more general problems with complex geometry and multiaxial stress states.

The aim of the present contribution is to assess the localization properties of nonlocal damage models under bending. To permit a partially analytical treatment and to display the fundamental properties of various formulations, the idealized case of uniform bending is considered. This means that the beam has a constant cross section and is subjected to a uniformly distributed bending moment and zero normal force. The beam is assumed to be very long, and boundary effects that might be introduced by the supports at the end sections are eliminated by imposing periodic boundary conditions on the fluctuating part of the response, which is superimposed on the solution that corresponds to uniform curvature. The average curvature serves as a loading parameter that is monotonically increased.

Under the described loading scenario, the solution initially remains uniform in each fiber parallel to the beam axis. The onset of localization is studied as a bifurcation into a nonuniform solution. The critical conditions at which such a bifurcation occurs and the characteristics of the localization pattern are investigated in detail, depending on the type of formulation and specific constitutive model and its parameters.

The localization process starts at the tensile face of the beam, which belongs to the boundary of the domain on which the problem is solved. Consequently, localization patterns are affected by the specific treatment of the boundary, i.e., by the boundary conditions for the implicit gradient-type formulation, and by the modification of the nonlocal weight function near the boundary for integral-type formulations. A comparison of various approaches is presented and the role of the boundary treatment is discussed.

In addition to the studies of the first bifurcation from the (fiber-wise) uniform solution, the subsequent evolution of the localized solution up to complete failure is explored by numerical simulations. Particular attention is paid to the dependence between the spacing and depth of active (growing) inelastic bands.

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REFERENCES

[1] G. Pijaudier-Cabot and Z. P. Bažant. Nonlocal damage theory. *Journal of Engineering Mechanics ASCE*, Vol. **113**, 1512–1533, 1987.

[2] E.C. Aifantis. On the microstructural origin of certain inelastic models. *Journal of Engineering Materials and Technology, ASME*, Vol. **106**, 326–330, 1984.

[3] R.H.J. Peerlings, R. de Borst, W.A.M. Brekelmans and J.H.P. de Vree. Gradient-enhanced damage for quasi-brittle materials. *International Journal for Numerical Methods in Engineering*, Vol. **39**, 3391–3403, 1996.

[4] Z.P. Bažant and M. Jirásek. Nonlocal integral formulations of plasticity and damage: Survey of progress. *Journal of Engineering Mechanics ASCE*, Vol. **128**, 1119–1149, 2002.

[5] R.H.J. Peerlings, M.G.D. Geers, R. de Borst and W.A.M. Brekelmans. A critical comparison of nonlocal and gradient-enhanced softening continua. *International Journal of Solids and Structures*, Vol. **38**, 7723–7746, 2001.

[6] M. Jirásek. Non-local damage mechanics with application to concrete. *Revue française de génie civil*, Vol. **8**, 683–707, 2004.