

# WALL-BASED FEEDBACK CONTROL OF AN INCOMPRESSIBLE LAMINAR BOUNDARY LAYER SUBJECTED TO FREE-STREAM VORTICAL DISTURBANCES

João Miguel da Rocha Pinto<sup>1</sup>, Pierre Ricco<sup>1</sup> and George Papadakis<sup>2</sup>

<sup>1</sup> Department of Mechanical Engineering, The University of Sheffield, Sheffield S1 3JD, UK,  
joao.pinto@sheffield.ac.uk, p.ricco@sheffield.ac.uk

<sup>2</sup> Department of Aeronautics, Imperial College London, SW7 2AZ, UK  
g.papadakis@imperial.ac.uk

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The laminar-turbulent boundary-layer transition induced by free-stream vortical disturbances of medium-to-high intensity is not induced by Tollmien-Schlichting waves, as predicted by classical stability theory, but involves the growth and breakdown of streamwise-elongated and low-frequency streaks, often called Klebanoff modes [3]. This type of transition is ubiquitous in many internal flows of technological and industrial scenarios, especially at high speeds, such as over gas turbine blades. It is usually referred to as bypass transition. The crucial feature is that the transition point usually moves upstream as the free-stream disturbance level becomes more intense, which has detrimental consequences for the efficiency of fluid systems because of the increase of wall friction drag. Therefore, delaying bypass transition is of utmost importance.

This paper presents theoretical and numerical results on the penetration of small-amplitude free-stream vortical disturbances into an incompressible laminar boundary layer, the formation and evolution of the Klebanoff modes inside the boundary layer and the wall-based feedback control of such disturbances. The theoretical formulation of the streaks follows the works of Leib, Wundrow & Goldstein [2], and is based on the incompressible linearised unsteady boundary region equations.

Adjoint control theory is applied to the equations of motion and considers small-amplitude wall transpiration actuation to attenuate the Klebanoff modes. This extends the work on spatial control by Cathalifaud & Luchini [1]. The major difference from this study is that free-stream turbulent forcing was not considered in [1], while here the influence of free-stream turbulence, which is the cause of the formation and downstream growth of the streaks, is rigorously accounted for. Matched asymptotic expansions are employed to

synthesize mathematically the mutual interaction between the outer free-stream vortical flow and the boundary layer and to arrive at the correct initial and boundary conditions for the governing equations.

Our results prove that Klebanoff modes can be substantially attenuated using blowing and suction while considering different free-stream vortical disturbances. The evolution of the maximum amplitude of the streamwise velocity fluctuation along the streamwise direction, with and without optimal actuation, is displayed in the left plot of figure 1. In the right plot of figure 1, the streamwise velocity profile, shown as function of the scaled wall-normal coordinate at a fixed streamwise location, presents two peaks when wall transpiration is applied. Our results differ from the ones of Cathalifaud & Luchini [1], who showed that full attenuation of the streamwise streaks can be achieved via optimal control based on wall transpiration. In our case, it is likely that optimal control leads to partial attenuation because of the presence of free-stream forcing.

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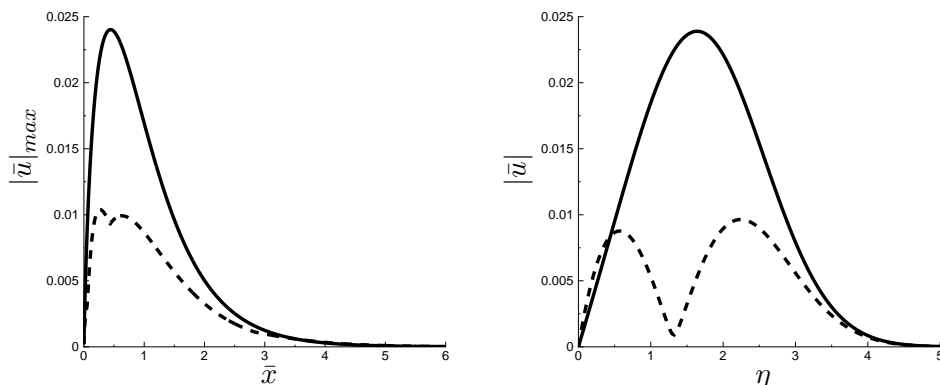


Figure 1: Comparison of the maximum amplitude of the streamwise velocity fluctuations (left) and of the wall-normal profiles of streamwise velocity fluctuations at  $\bar{x} = 2\pi x/\lambda_x$  (right) without (solid) and with (dashed) optimal wall transpiration for  $\kappa = \sqrt{2\pi\nu\lambda_x/U_\infty}/\lambda_z = 1$ ,  $\kappa_2 = \sqrt{2\pi\nu\lambda_x/U_\infty}/\lambda_y = -1$ , where  $\nu$  is the kinematic viscosity,  $U_\infty$  is the mean free-stream velocity, and  $\lambda_x$ ,  $\lambda_y$ , and  $\lambda_z$  are the wavelengths of the free-stream disturbance along the Cartesian directions.

## REFERENCES

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